1. First century of flight: ballooning

Before the Wright Brothers flew their first aircraft, people used balloons to fly. However, since their large volume their drag is too big for commercial flights to become true. Long time ago, Zhuge Liang used Kong-Ming lanterns as hot air balloons. The Montgolfier brothers were the first to move humans onto a balloon in the late 18th century. Pilatre de Rozier was the first to complete it, and is widely recognized as a flight pioneer.

The equation of state relates pressure with density and temperature. Derivation:

$$V = \frac{m}{\rho} \rightarrow pV = n\Re T$$

$$p\frac{m}{\rho} = \frac{m}{M}\Re T \rightarrow p = \rho\frac{\Re}{M}T = \rho RT, where\ R_{air} = \frac{\Re}{M_{air}}$$

R_{air} is only valid for air. If the gas is the same,

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$

Archimedes, a famous physicist, concluded that, because a block in water is in equilibrium, there must be a buoyancy force upwards of exactly the same magnitude as the weight of the water. This principle is used for balloons. If there was no special gas in the balloon, there is equilibrium. However, if a less dense gas is in the balloon, the weight of that balloon is smaller than if there was air inside it. This generates lift:

$$L = W_{air} - W_{gas} = \rho_{atm}gV - \rho_{gas}gV = \rho_{atm}Vg\left(1 - \frac{\rho_{gas}}{\rho_{atm}}\right)$$

This formula applies to hot air balloons (less dense) and gas air balloons.

Lift for hot air balloons:

$$L = \rho_{atm}Vg\left(1 - \frac{\frac{p}{RT_{hot}}}{\frac{p}{R}T_{atm}}\right) = \rho_{atm}Vg\left(1 - \frac{T_{atm}}{T_{hot}}\right) = \rho_{atm}Vg\frac{\Delta T}{T_{atm} + \Delta T}$$

Use Kelvins for temperature. Pressure inside the balloon is equal to pressure of the atmosphere. When the altitude is that high that p_{atm} decreases, V becomes larger to allow a bigger volume so that p_{inside} also becomes smaller ($p = \frac{RT}{\rho}$), until the pressure difference is too high that the balloon will explode. To calculate the mass the mass of the balloon, use $m_{tot}g = L$. Gas balloons:

$$L = \rho_{atm}Vg\left(1 - \frac{M_{gas}}{M_{air}}\right)$$

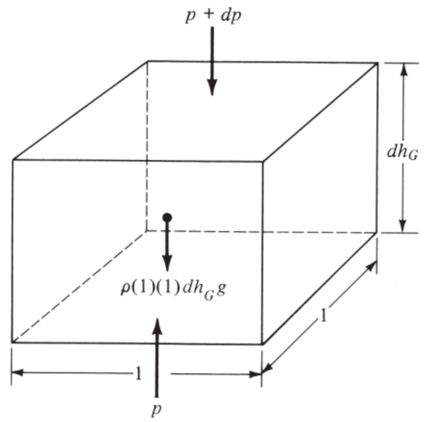
To get a feeling which one is better, $\left(1 - \frac{M_{gas}}{M_{air}}\right) = \frac{6}{7}$, this means $L = \frac{6}{7}W_{air}$, to get the same lift for hot air, $\frac{\Delta T}{T_{atm}+\Delta T} = \frac{6}{7} \rightarrow \Delta T = 6T_{atm}$. This means gas balloons are much more efficient. If a balloon is stationary for a long time, the gas inside the balloon will slowly dissipate out the balloon and is replaced with air.

2. The standard Atmosphere

The absolute altitude is the geometric altitude above sea level plus the radius of the earth, $h_a = h_G + r_E$. This will affect gravity.

$$\frac{g(h_G)}{g_0} = \frac{GM}{(r+h_G)^2} \cdot \frac{r^2}{GM} \rightarrow g(h) = g_0 \left(\frac{r}{r+h_G}\right)^2$$

As the altitude increases, the temperature, pressure and density of air varies. The standard atmosphere model can be used to calculate these variables at different heights. Consider a box of air with height dh.



Since it is stationary, there is pressure on top, pressure on the bottom and weight. These are balanced, we can obtain the hydrostatic equation,

$$pA = (p + dp)A + mg \rightarrow pA = pA + Adp + \rho A\, dhg$$

$$dp = -\rho g dh$$

This relates pressure and height difference, applicable to fluids as well. Using $p = \rho RT$, you can express the following model for T=constant, called the isothermal layers. When calculating with temperatures, look if the temperature is constant between the altitudes in a h, T diagram.

$$\frac{dp}{p} = -\frac{g}{RT} dh \rightarrow \frac{p_1}{p_0} = e^{-\frac{g}{RT}(h_1 - h_0)} = \frac{\rho_1}{\rho_0}$$

For a changing T, T increases linearly as height increases, $T = T_0 + a(h - h_0)$, where a is the gradient of T with respect to h. For different regions within our atmosphere, T changes with h differently. This pattern of alternating temperature gradients is primarily due to the varying composition and density of the atmosphere, as well as how different layers absorb solar radiation. $\frac{dT}{dh} = a$, so

$$\frac{dp}{p} = -\frac{g}{RT} dh \rightarrow \frac{dp}{p} = -\frac{g}{Ra}\frac{1}{T} dT$$

$$\frac{p_1}{p_0} = \left(\frac{T_1}{T_0}\right)^{-\frac{g}{Ra}}, , where\ p = \rho RT\ and\ T = T_0 + a(h - h_0)$$

Be aware of different layers within the atmosphere. The zero conditions when calculating new variables at a different height must have the same corresponding a. To calculate this, calculate these zero conditions for each beginning of the layer. Convenient way of checking, density and pressure always decreases as altitude increases. So, the pressure, temperature, and density at a certain altitude can all be expressed for the standards at sea level. The foundations of these equations are: $p = \rho RT$, $dp = -\rho g dh$, and $T_f = T_i + a(h_f - h_i)$. To let g be constant, we define a geopotential height h, the hydrostatic equation uses the geopotential height. It is fictious, and differs from the geometric height h_G. To account for a same change in pressure dp,

$$-\rho g_0 dh = -\rho g dh_G$$

$$dh = \frac{g}{g_0} dh_G = \frac{R_E^2}{(R_e + h_G)^2} dh_G$$

Solving this, using integration between sea level and their heights, gives

$$h = \frac{r_e}{r_e + h_g} h_G$$

You can see that the geopotential height is smaller than geometric height. This is explained below.

In $-\rho g dh_G$, as h_G increases, g decreases, and in $-\rho g_0 dh$, $g_0 = cons$, meaning that if $dh_G = dh$, $|-\rho g dh_G| < |-\rho g_0 dh|$. However, both expressions must be equal to account for same change in pressure. This means that dh will be smaller than dh_g to account for same dp.

When solving problems, the following you should know by heart: ρ_0 = 1.225 kg/m3, T = 288.15 K (15 C), p = 1.01325· 10^5 Pa.

3. Forces on an airplane

Since mass can neither be destroyed nor created, the mass flow (mass displaced per unit time) is constant along a streamline. Since $\frac{dm}{dt} = \rho dv = \rho A \frac{dx}{dt} = \rho AV$, So,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Incompressible flows can be assumed for low airspeeds (since the dynamic pressure will be smaller, so volume and mass can be assumed constant at one point). For incompressible flows,

$$A_1 V_1 = A_2 V_2$$

Bernoulli's equation relates total pressure with static and dynamic pressure.

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

If you consider an infinitesimal box of air moving horizontally, then $ma = pA - (p + dp)A$. A is the area swept, so $A = v * dx$, dx is the distance swept. Pressure is a point property, and acts always perpendicularly to the object (molecules have momentum and transfer energy per unit area), and shear stress is the movement of molecules along the surface (tangentially). The air is pushed at the side where the force is pA, so the velocity and the dynamic pressure is bigger, hence $dp < 0$. In Bernoulli's equation, frictional shear and gravity are neglected, hence the movement in the vertical direction is neglected.

Important:

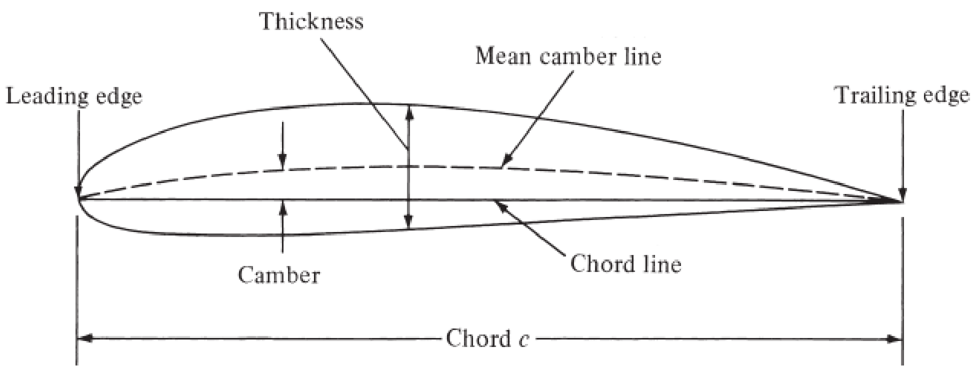
1. Only for inviscid (frictionless) incompressible flow
2. Different points along streamline,
3. Density is constant (incompressible), so low Mach number.

If you have a flow of air, then use Bernoulli's equation, if you have a point in the air, then use the equation of state.

Reynold's number tells how smooth the flow is.

$$Re = \frac{\rho V x}{\mu}$$

Applicable to both fluids and gases, however as temperature increases, air gets thicker ($\rho = pRT$), but fluids get thinner (difference!). μ is the viscosity (how much friction it causes). If the density and velocity is high and can move freely (μ is low) and the distance is high (x), then the motion will get more irregular. So the higher the Reynolds number, the more turbulent the flow is. For a wing, x is the chord length of the wing.



Camber is the maximum distance between the camber line and the chord line. Lift is in perpendicular to drag, drag in the direction of wind speed.

$$L = \frac{1}{2}\rho V^2 S C_L$$

$$D = \frac{1}{2}\rho V^2 S C_D$$

Where C_L and C_D are a function of the angle of attack, airfoil and wing features. This means that lift is generated at the expense of drag (pressure difference creates both lift and drag).

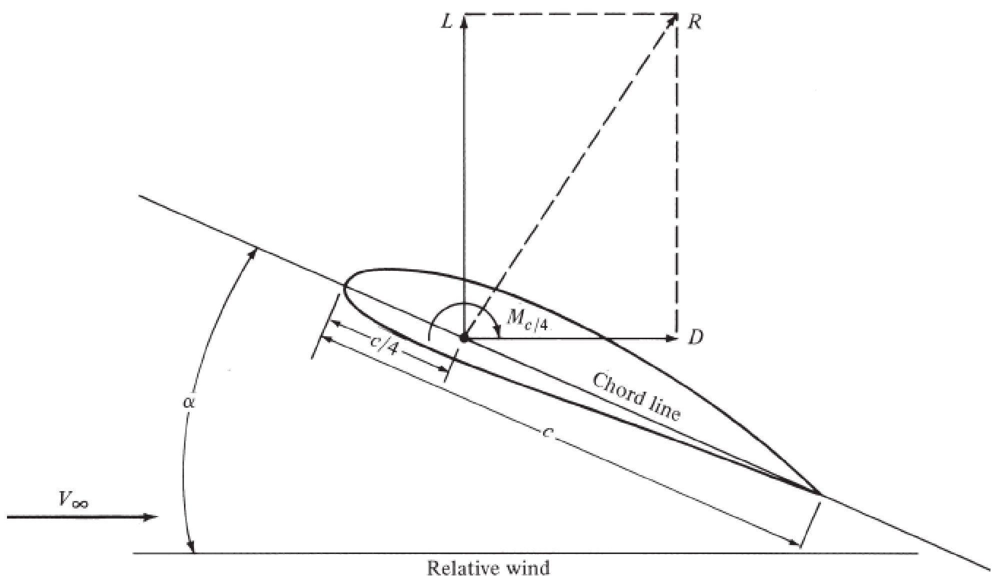
Drag consists of induced drag and parasitic drag. Induced drag is caused by the lift, that is, the pressure differences cause vortices. Parasitic drag consists of the shear stress of the flow along the surface.

$$C_D = C_{D0} + kC_L^2 = C_{D0} + \frac{C_L^2}{\pi A e}$$

C_{D0} is the parasitic drag, and the other term is the lift-induced drag. The glide ratio is the ratio between L/D and thus C_L/C_D. This ratio needs to be maximum. The induced drag involves the aspect ratio A, $A = \frac{b^2}{S} = \frac{b}{c}$. Making the wingspan longer will result in less drag, however the drawback of this is that it will be difficult to maneuver the aircraft and the strength of the wing will go down.

An airplane flies because the static pressure on top of the airfoil is smaller than at the bottom, because the velocity is higher, due to it can be assumed as a narrow tube, and the mass flow is constant.

NACA0015, first digit is the camber as a percent of the chord line, Second digit describing the distance of maximum camber from the airfoil leading edge in tenths of the chord, last digits the max thickness as a percent of the chord line. NACA0015 is symmetrical, as the camber line is in line with the chord line.



As the wing is put under an angle of attack, there is more airflow on the bottom so a higher static pressure. The lift coefficient increases linearly, until the maximum is hit, where the flow of air is unattached at the top of the airfoil An airfoil with camber will create lift even if $a = 0$, because the camber line already creates pressure difference. A symmetric airfoil will not do this.

To look at a single region within a wing, the lift per unit span is the lift generated where the wing surface area S = $c(1) = c$. This means that the lift per unit span is

$$L = \frac{1}{2}\rho V^2 c C_L$$

In 2D wing analysis, the lift per unit span is calculated and is assumed to be constant over the whole wing (without wing tips). When obtaining C_L and C_D for a given airfoil, first calculate Reynolds number to obtain the designated graph.

The stalling speed is the lowest speed at which the airplane can fly steadily, this is attained when the lift coefficient is the biggest.

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,max}}}$$

However, in the horizontal flight, C_L is already fixed from the geometry. To increase C_L, flaps are used. They increase the camber line so a bigger lift coefficient is created and a bigger wing surface area. This is handy at low airspeeds, to still create sufficient lifts (take off or landing). However, during take off, the aircraft is accelerating so drag should be minimal. That's why the flaps won't be extended fully during take off, since drag plays a role.

Other explanations for generating lift:

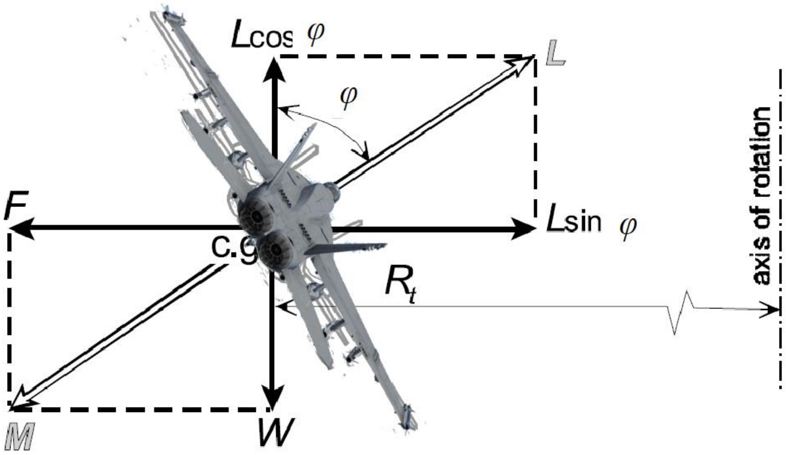
- Longer pathway, the air on top needs a longer path so more velocity is needed to meet at the same time (no law states that this should be the case).
- Air is pushed down, as air meets the bottom of the air foil, the angle off attack causes it to be pushed down, which causes an opposite reaction force up.
- Circular lift. As air moves circularly around the airfoil, the velocity on the bottom is higher than on top.

The main explanation is that a pressure difference creates lift.

Snowball-effect: small weight → less lift → less fuel → small weight

4. Stability and control

If an airplane makes a turn, the lift will be banked at a bank angle ϕ. If the airplane is in equilibrium in the vertical direction, then $L\cos(\phi) = W$. Thus, $\boldsymbol{F_R} = \boldsymbol{L} + \boldsymbol{W}$, this resultant force is the centripetal force to keep the circular motion.

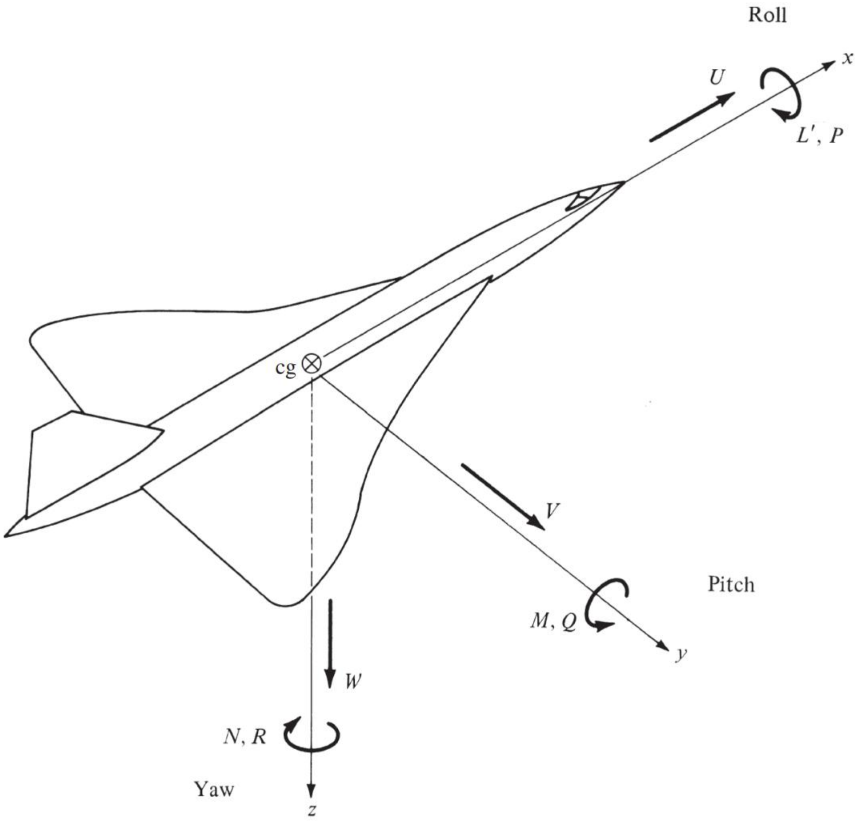


The load factor n is how much of lift the aircraft is experiencing relative to its weight, $n = \frac{L}{W} = \frac{L}{L\cos(\phi)} = \frac{1}{\cos(\phi)}$. To make the turning as fast as possible the resultant force needs to be big, so a large centripetal acceleration is operating. Since $F_r = L\sin\phi$, lift needs to as big as possible, and thus the load factor needs to be big. Also, to make the turning as big as possible (the bending is strong), the velocity needs to be small. Since the velocity is defined as the tangent to the circular path, the smaller this velocity is, the easier the vector can be ‘bent’.

$$\tan\phi = \frac{V^2}{gR_T}$$

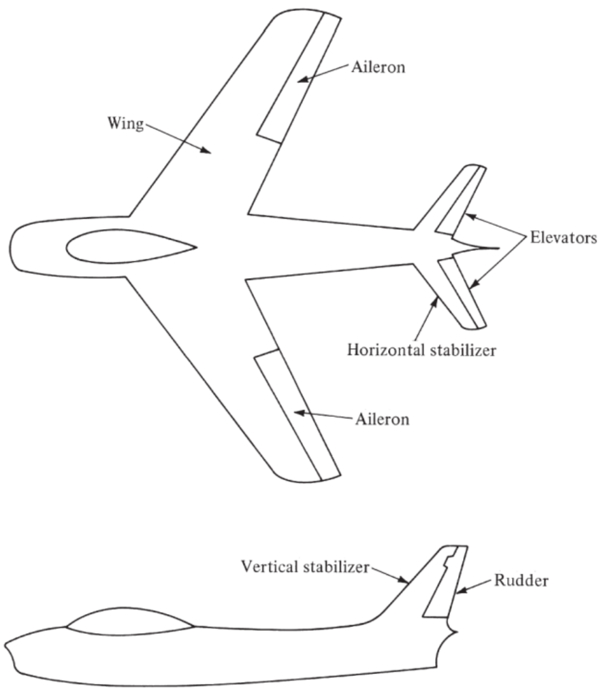
Where R_t is the distance between the c.g. and the axis of ration.

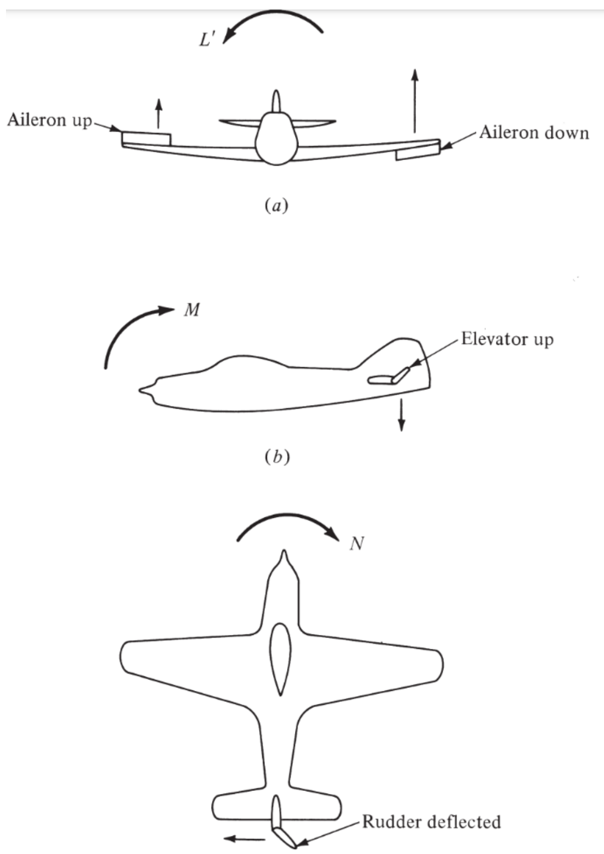
Stability of an aircraft involves (sudden) moments that change the rotation of the airplane while in flight.



Rotation along the z axis is called yawing, and N is the yaw moment. Rotation along the x axis is rolling, and L’ is the rolling moment. Along the y axis is called pitching, and M is the pitching moment. To control these moments, controls are added to withstand these rotational moments.

- Ailerons are mounted at the trailing edge of the wing, near the wing tips. In Fig. 7.4, one aileron is deflectd up and the other down, creating a differential lifting force, thus contributing and controlling the moment L'. Rolling is called lateral motion. Ailerons are lateral controls.
- The elevators are located on the horizontal stabilizer. In Fig. 7.4 the elevator is deflected upward, which causes a negative lift at the tail and thus contributing to the pitching moment M. The moment which causes the plane to be pushed downwards is called longitudinal motion. Elevators are longitudinal controls.
- The rudder is deflected to the right, creating a leftward aerodynamic force on the tail and thus contributing to the yawing moment N. Yawing is called directional motion.





Rolling is called lateral motion, and the ailerons are the lateral controls. Pitching is called longitudinal motion, and elevators are longitudinal controls. Rudders control directional motion, hence the rudder is a directional control.

Any disturbance by, for example, the wind or weather, changes the stability of an airplane.

If the forces and moments on the body caused by a disturbance tend to initially to return the body toward its equilibrium position, the body is statically stable. The body has positive static ability. If the forces and moments are such that the body continues to move away from its equilibrium position after being disturbed, the body is statically unstable. It has negative static stability.

If the moments and forces add up to zero and does not change due to disturbances, the system is neutrally stable.

The form of stability is static stability.

The moment of the aerodynamic force can be calculated using

$$M = \frac{1}{2}\rho V^2 S c C_m$$

For longitudinal,

$$M_{Long} = \frac{1}{2}\rho V^2 S c C_m$$

For lateral,

$$M_{lat} = \frac{1}{2}\rho V^2 SbC_{Roll}$$

When all of the moments at the center of gravity add up zo zero, it is in static equilibrium.
Now consider an airplane flying at an α_e in which the moments are the center of gravity are zero, if it is in equilibrium, this is called the trim angle of attack, or α_e. If there is a disturbance causes the angle of attack to change to α.

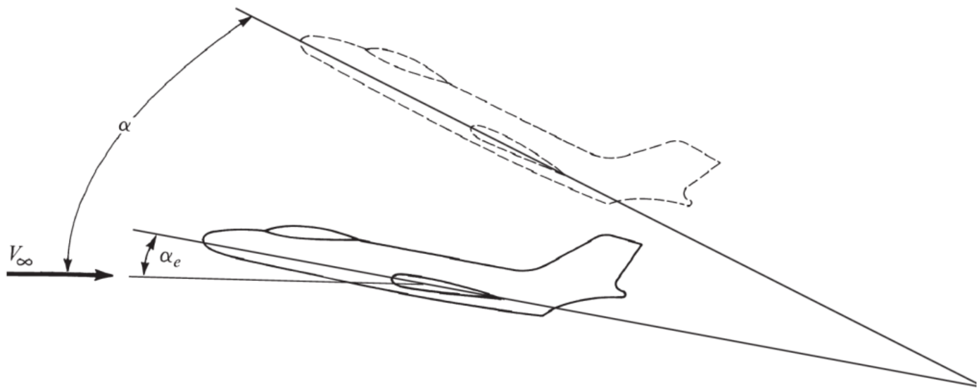
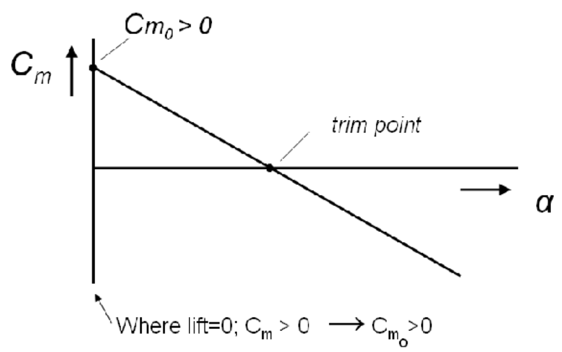


Figure 7.7 Disturbance from the equilibrium angle of attack.

From, to regain stability, you will need a change in moment acting in opposite direction to the angle attack. Thus, for stability, we need a *negative* change of the pitching moment if there is a positive change of the angle of attack (and vice verse), so:

$$\frac{dC_m}{d\alpha} < 0$$

Graphically, this means that the steepness of the graph must be negative. On the other hand, it needs to have a positive starting point to get a trim angle of attack. When $C_m = 0$, the moments are zero (trim point).

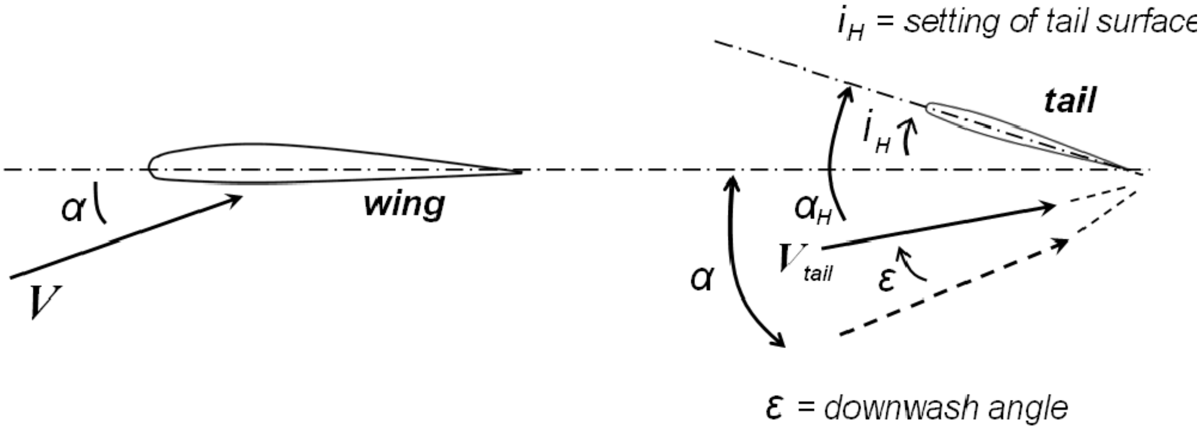


So, two conditions for stability:

- $C_{m_0} > 0$; if lift = 0, there still needs to be a pitching moment to not fall to the ground.
- $\frac{dC_m}{d\alpha} < 0$.

The forces that govern this moment are the lift force produced by the wing and tail. To look at condition 2, we must form an expression to relate the change in pitch moment with the angle of attack. To do that, we must find what happens to the tail when the angle of attack changes:

0



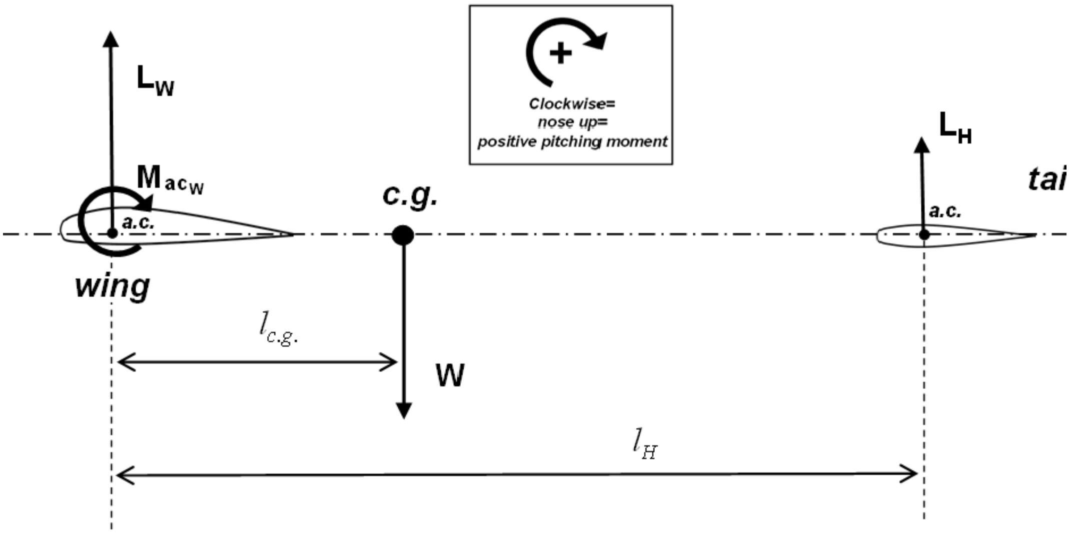
The wing has pushed air down and decreased the angle of attack with $\varepsilon = d\alpha$. So the tail angle of attack becomes:

$$\alpha_H = \alpha - \varepsilon + i_H$$

Now, the change in α_H due to a change in angle of attack is:

$$\frac{d\alpha_H}{d\alpha} = \frac{d}{d\alpha}(\alpha - \varepsilon + i_H) = 1 - \frac{d\varepsilon}{d\alpha}$$

$\frac{d\varepsilon}{d\alpha}$ means the change in downwash due to the change in angle of attack. After defining the tail surface area. Look at the free body diagram:



Here, $M_{a.c}$ is moment produced at the aerodynamic center: the point where the moment does not change when the angle of attack changes, $C_{m,ac} = const$ with different angles of attacks. Keep in moment due to pressure deflection, the moment points are everywhere, but the moment at a.c. stays constant.

Thus, the sum of the moments at tail and wing, are:

$$M = M_{a.c.wing} + L_W l_{cg} - L_H(l_H - l_{cg})$$

Note that L_H can be directed upward or downward.

After some simplifications:

$$M = M_{a.c.wing} + L_W l_{cg} - L_H\left(l_H - l_{cg}\right)$$

$$M = M_{a.c.wing} + (L_W + L_H)l_{cd} - L_H l_h = M_{a.c.wing} + L\, l_{cg} - L_H l_h$$

Converting this to moment coefficients so that the equation becomes dimensionless:

$$\frac{M}{\frac{1}{2}\rho V^2 Sc} = \frac{M_{a.c.wing}}{\frac{1}{2}\rho V^2 Sc} + \frac{L l_{cg}}{\frac{1}{2}\rho V^2 Sc} - \frac{L_H l_H}{\frac{1}{2}\rho V^2 Sc}$$

$$C_M = C_{m_{ac_w}} + C_L \frac{l_{cg}}{c} - C_{L_H} \frac{\frac{1}{2}\rho V^2 S_H l_H}{\frac{1}{2}\rho V^2 Sc}$$

$$C_M = C_{m_{ac_w}} + C_L \frac{l_{cg}}{c} - C_{L_H} \frac{S_H l_H}{Sc} = C_{m_{ac_w}} + C_L \frac{l_{cg}}{c} - C_{L_H} V_H$$

$\frac{S_H l_H}{Sc}$ can be called the tail volume V_H. Applying the derivative

$$\frac{dC_M}{d\alpha} = \frac{dC_{M_{ac_w}}}{d\alpha} + \frac{dC_L}{d\alpha}\frac{l_{cg}}{c} - \frac{dC_{L_H}}{d\alpha} V_H$$

Note that at the aerodynamic center $\frac{dC_{M_{ac_w}}}{d\alpha} = 0$, so

$$\frac{dC_M}{d\alpha} = \frac{dC_L}{d\alpha}\frac{l_{cg}}{c} - \frac{dC_{L_H}}{d\alpha} V_H$$

$\frac{dC_L}{d\alpha}$ is simply the steepness of a C_L, α-graph, similar to $\frac{dC_L}{d\alpha_H}$. But in the equation, α is not a_H, but

$$\frac{dC_L}{d\alpha} = \frac{dC_L}{d\alpha_H}\frac{d\alpha_H}{d\alpha}$$

$$\frac{dC_L}{d\alpha} = \frac{dC_L}{d\alpha_H}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

Thus,

$$\frac{dC_M}{d\alpha} = \frac{dC_L}{d\alpha}\frac{l_{cg}}{c} - \frac{dC_{L_H}}{d\alpha_H}\left(1 - \frac{d\varepsilon}{d\alpha}\right) V_H$$

Now we can express $\frac{dC_L}{d\alpha}$ with $\frac{dC_L}{d\alpha_H}$ the corresponding steepness a and a_H of the curve. Therefore,

$$\frac{dC_M}{d\alpha} = a\frac{l_{cg}}{c} - a_H\left(1 - \frac{d\varepsilon}{d\alpha}\right) V_H < 0$$

$$a_H\left(1-\frac{d\varepsilon}{d\alpha}\right)V_H > a\frac{l_{cg}}{c}$$

With $V_H = \frac{S_H l_H}{Sc}$

From this,

- A larger tail will contribute to static stability
- A longer distance between tail and wing will contribute to stability.
- A c.g that is just after the wing or even before the wing contributes to stability.

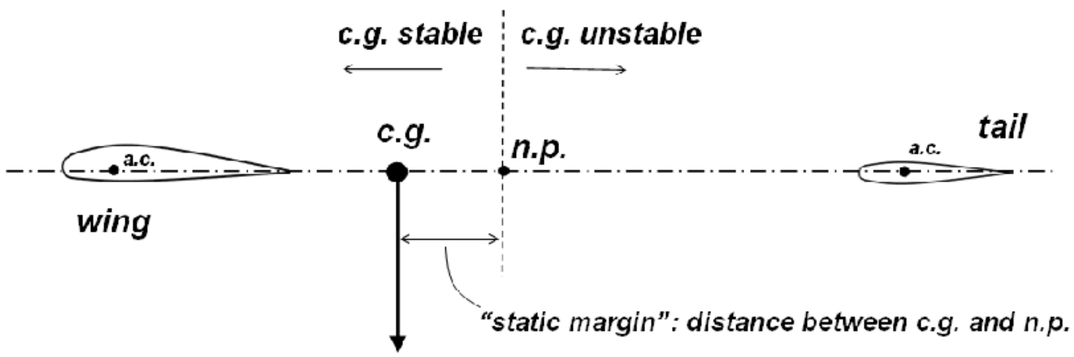
Recall

$$a\frac{l_{cg}}{c} - a_H\left(1-\frac{d\varepsilon}{d\alpha}\right)V_H < 0$$

This is a condition for stability, so the minimum length between center of gravity and the wing is when $a\frac{l_{cg}}{c} - a_H\left(1-\frac{d\varepsilon}{d\alpha}\right)V_H = 0$, in this case the length is called the neutral point, or l_{np}. Any center of gravity closer to the wing than the neutral will provide longitudinal stability. This minimum length is:

$$\frac{l_{np}}{c} = \frac{a_H}{a}V_H\left(1-\frac{d\varepsilon}{d\alpha}\right),\ V_H = \frac{S_H l_H}{S\cdot c}$$

Therefore, the neutral point is the aerodynamic center of the entire airplane, since the moment doesn't change at the aerodynamic center of this, $\frac{dC_m}{da} = 0$ if you assume this.



The static margin is the distance between center of gravity and neutral point:

$$l_{n.p} - l_{c.g}$$

- $C_{L_H} = a_H \cdot \alpha_H = a_H(\alpha - \varepsilon + i_H)$
- $\frac{dC_L}{d\alpha}, \frac{dC_L}{d\alpha_H}$ are both linear relationships, so use $\frac{\Delta C_L}{\Delta\alpha}, \frac{\Delta C_L}{\Delta\alpha_H}$
- If the airplane were to lose its tail, then the airplane can only produce the moment by the wing, directed upwards. That means when the plane is disturbed and flown upwards, there would not be a stabilizer.

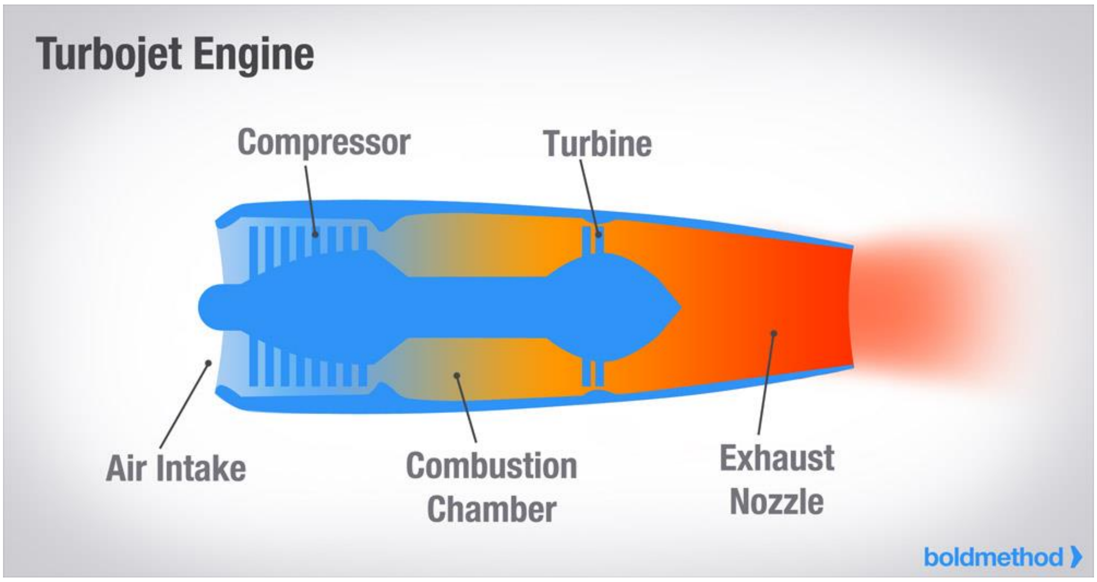
- A canard is small-sectional wing located in front the main wing, hence causing a moment upward. Because it is located in the same line of action, the canard will not cause a downwash. After some calculation, it can be found that,

$$\frac{l_{cg}}{c} > \frac{\left(\frac{dC_L}{d\alpha}\right)}{\left(\frac{dC_{L_{canard}}}{d\alpha}\right)} V_c = \frac{\left(\frac{dC_L}{d\alpha}\right)}{\left(\frac{dC_{L_{canard}}}{d\alpha}\right)} \frac{S_c l_c}{Sc}$$

The difference for this condition, is that $\frac{l_{cg}}{c}$ must be greater and not smaller than expression on the right. Hence, the smaller area and distance between canard and wing is, the more the plane will provide more ability. Therefore, the function of a canard is because it looks 'cool'.

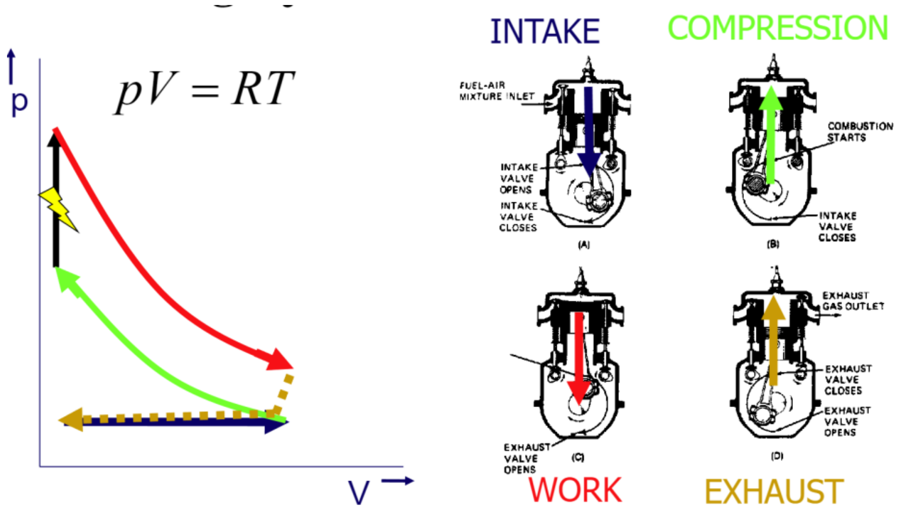
5. Propulsion

In a jet engine,



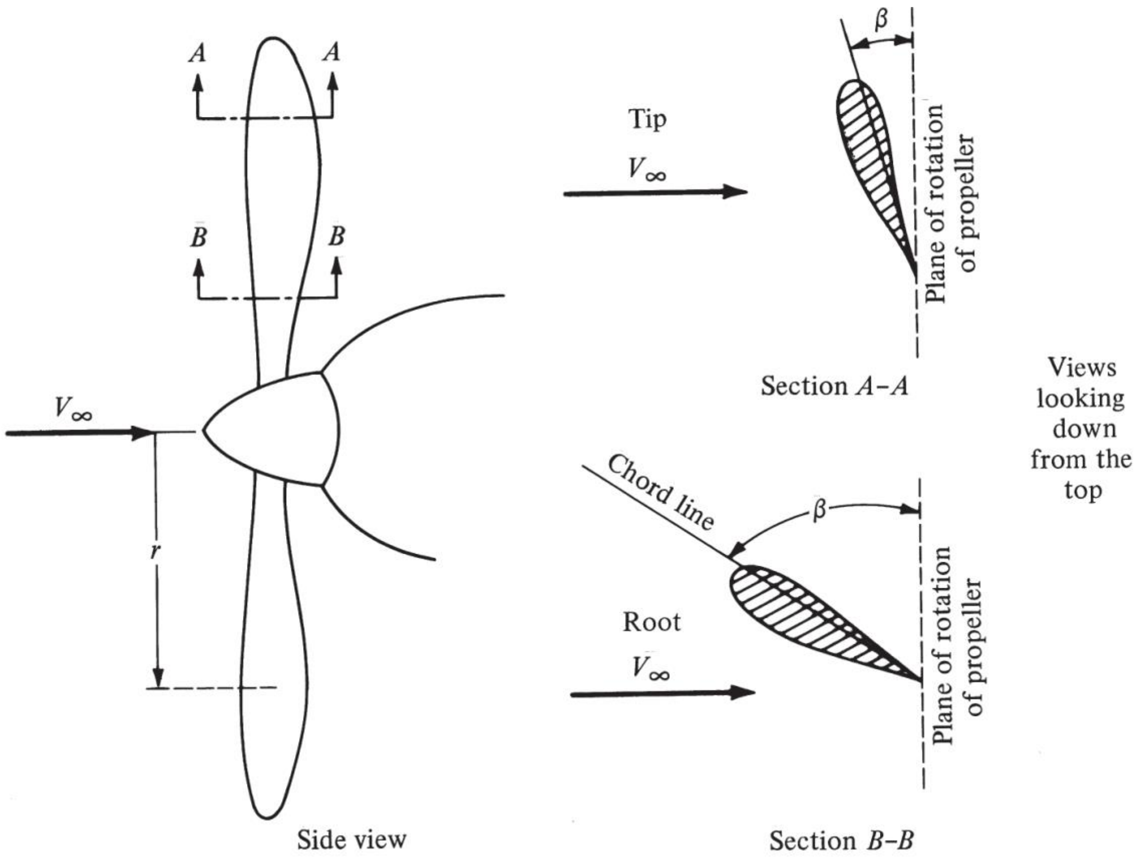
The intake compresses the incoming air, slightly increasing its pressure before it enters the compressor stage. Each stage progressively compresses the air. The high-pressure air is then directed into the combustion chamber. This process increases the temperature and pressure of the air. The fuel (usually kerosene) is injected into the compressed air and ignited. This combustion process releases a large amount of heat energy, which causes the gases to expand rapidly. The pressure inside the combustion chamber stays roughly constant, but the temperature rises significantly. The hot, high-pressure gases pass through a series of turbine blades. These blades rotate and transfer energy to the compressor via a shaft. This energy is used to continue compressing the incoming air. Only a portion of the energy is extracted by the turbine, while the remaining energy drives the exhaust gases through the nozzle to produce thrust. The remaining high-velocity gases exit the engine through the nozzle. As the gases expand and are expelled at high speeds, they create a powerful thrust force that pushes the engine and the aircraft forward.

In a piston engine, the incoming air expands the volume. Afterwards the air is compressed and temperature is added. The expansion releases the energy from compression + the energy from the heating, which came from igniting fuel. This creates a net force which can turn a propeller for example.



The mass flow through the engine stays constant, however, the velocities within the engine can change. The airspeed is the incoming speed V_0, and the gases leave the nozzle with jet velocity V_J. $T = m\frac{\Delta V}{\Delta t} = \dot{m}(V_J - V_0) = \rho A V_0 (V_J - V_0)$. When the airspeed increases, ΔV decreases, but the mass flow again increases. This means that T is constant with a turbojet engine, and $P_a = TV$.

A propeller engine uses either a piston engine or a turbo engine to rotate a propeller. In both cases, the engine generates power that turns the propeller, which moves air to create thrust (Newton's third law). The thrust generated by the propeller pushes the aircraft forward through the air.



The pitch angle is the angle β and is the angle between the airfoil of the blade and the vertical axis. For a larger pitch angle, the thrust is less, because less air is pushed. For a smaller pitch angle, the thrust is more, because more air is pushed. The shaft power is the power for the propellers, which as a result produces the mechanical power available, $P_a = TV_0$. Since the pitch can vary, power available can become constant for different values of airspeeds. So, $T = \frac{P_a}{V_0}$. The propeller efficiency is,

$$\eta_{prop} = \frac{P_a}{P_{br}} = \frac{TV_0}{P_{br}}$$

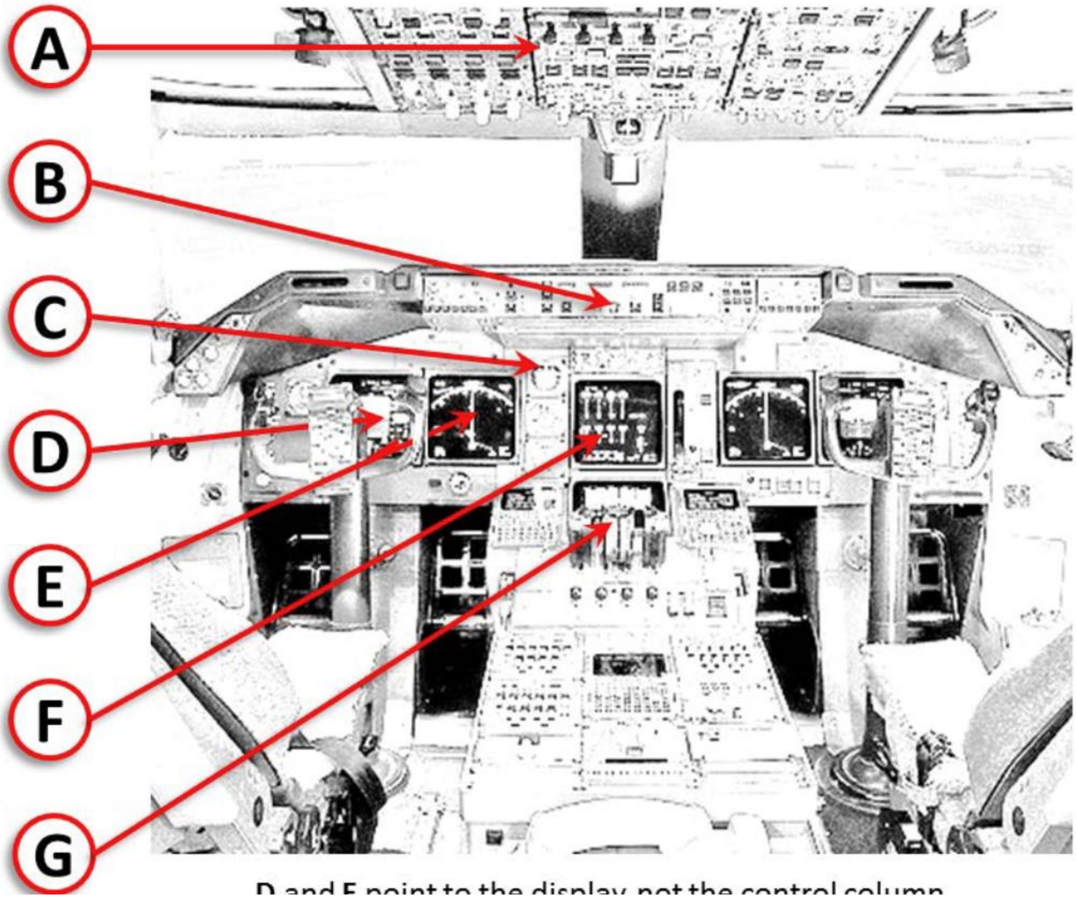
For a jet engine,

$$\eta_{jet} = \frac{P_a}{P_{br}} = \frac{TV_0}{\frac{1}{2}\dot{m}V_J^2 - \frac{1}{2}\dot{m}V_0^2} = \frac{2\dot{m}(V_j - V_0)V_0}{\dot{m}(V_j + V_0)(V_j - V_0)} = \frac{2V_0}{V_J + V_0} = \frac{2}{1 + \frac{V_j}{V_0}}$$

A jet engine moves small amounts of air really fast. At low airspeeds, this creates more drag and thrust created. In contrast, propellers move a large volume of air at low speed to create thrust. So, at low airspeeds propellers are more efficient. This difference also demonstrates the bypass ratio, $B = \frac{\dot{m}_{cold}}{\dot{m}_{warm}}$. Having a large amount of air at a release velocity (more air but less air to ignite to increase the release velocity) is more efficient. So, large bypass engines are used for commercial aircraft (lower speed), and low bypass engines for military jet fighters.

6. Cockpits and systems

Here are the different elements of a cockpit of a Boeing 737.

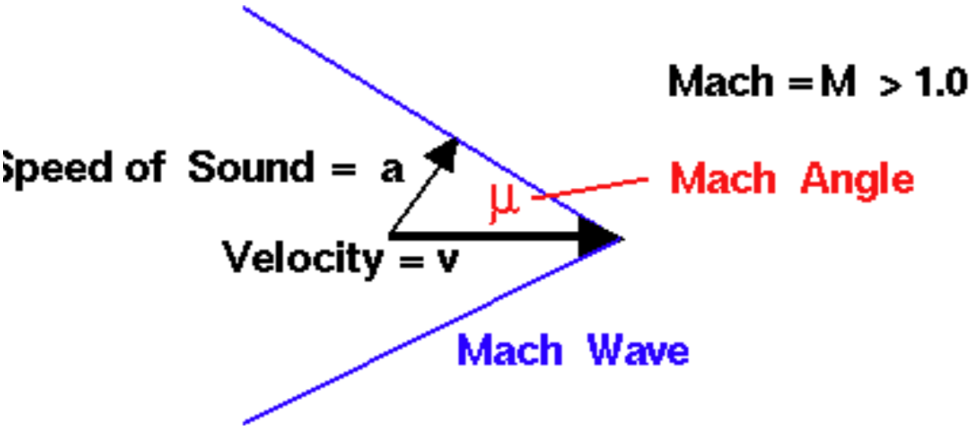


A. **Overhead Panel** – Control systems of all the parts of the aircraft.
B. **Mode Control Panel or Flight Mode Panel or Autopilot panel** – Control autopilot
settings, for example speed, Mach number.
C. **Standby instruments (Artificial Horizon)** – View attitude of plane in case of e.g. power
failure
D. **Primary Flight Display** – View attitude, speed and altitude of aircraft (also A/P mode)
E. **Navigation Display(or HSI, horizon situation indicator**) – View compass, direction
and/or map for nav purposes.
F. **Engine Indicators/Instruments (Eng Ind & Crew Alertin System) EICAS** – Check
status of engines and other systems
G. **Throttle Levers** – Control engines

Mach number relates the flow velocity with the speed of sound, $M = \frac{V_f}{a}$. Where a is the speed of sound. Since the higher the altitude, the lower the temperature of the air. This means that the speed of sound is not constant for every altitude. $a = \sqrt{\gamma RT}$, where $\gamma = 1.4$ for air and $R = 287$ for air and T is temperature. $M = \frac{V_{TAS}}{\sqrt{\gamma RT}}$.

The temperature can be calculated using ISA. Which means that speed of sound is a function of altitude.

When the Mach number is larger than 1, the aircraft experiences supersonic flight. As the airspeed exceeds the speed of sound, shock waves are created. These shock waves deflect the direction of the speed of sound with an angle, known as the Mach angle.



From this, the mach angle can be calculated as follows:

$$\cos\mu = \frac{a}{V} = \frac{1}{M}$$

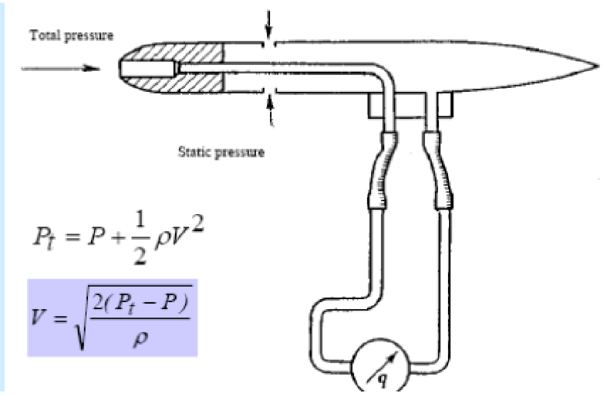
Airspeed is the speed of the plane without the influence of wind. The groundspeed is the airspeed + the influence of the wind. The true airspeed is the airspeed at which the density of the air at that height is taken. The equivalent airspeed is the airspeed at which the density of the air at sea level is taken. So,

$$\frac{V_{Eq}}{V_{true}} = \frac{\sqrt{\frac{2(p_t - p_s)}{\rho_0}}}{\sqrt{\frac{2(p_t - p_s)}{\rho(h)}}} = \sqrt{\frac{\rho(h)}{\rho_0}}$$

Since the equivalent airspeed is calculated by relating the pressure difference at sea level conditions, the performance of the aircraft can be determined using the equivalent airspeed.

The static pressure, is a result of intermolecular reactions within the moving air (relative speed = 0) (perpendicular to the flow), and the total pressure is when an external object experiences both the speed and the intermolecular reactions, so it is the p_{atm} at that altitude, using ISA. The difference is the dynamic pressure. The total pressure is the pressure that would exist if the flow were slowed down to zero velocity. A pitot tube measures the static pressure by having an opening perpendicular to the flow, and the total pressure by slowing it down to V=0, the difference gives dynamic pressure and hence V can be calculated.

$$V_{TAS} = \sqrt{\frac{2q}{\rho}} = \sqrt{\frac{2(p_{tot} - p_{st})}{\rho}}$$



$$p_{total} = p_{static} + p_{dynamic}$$

$$p_{total} = p_{static} + q$$

$$p_{total} = p_{static} + \frac{1}{2}\rho V^2$$

$$p_{total} = p_{static} + \frac{1}{2}\rho_0 V_{EAS}{}^2$$

The vertical speed, which is a measurement of ascents or descends, can be measured by measuring the rate of change in the static pressure. Because the static pressure is an indicator of altitude, the rate of change in static pressure gives you the rate of change in altitude, or vertical speed.

There is a problem, because the density varies with altitude. And because airplane cruise irregularly at different altitudes, the true airspeed must be changed to maintain the dynamic pressure. The dynamic pressure is how the aircraft
'feels' the speed. When you go higher, the true airspeed must be increased to get the same 'feeling'.

The convenience of the equivalent airspeed is that it is an indicator of the value for the dynamic pressure, as

$$\frac{1}{2}\rho_0 V_{EAS}^2 = \frac{1}{2}\rho_h V_{TAS}^2 = q$$

This way, when you know the equivalent airspeed, the dynamic pressure can instantly be calculated and independent of altitude.

An alternative is to increase the lift coefficient by increasing the angle of attack. When it hits the stalling point, $C_{L_{stall}} = C_{L_{max}}$. The maximum lift coefficient gives you the stall speed V_{stall}, which is also the minimum airspeed V_{min}.

$$V_{stall} = \sqrt{\frac{W}{S}\frac{2}{\rho(h)}\frac{1}{C_{L_{max}}}}$$

Notice the density dependent on air. So the aircraft will constantly experience different stall speeds. Using the equivalent airspeed:

$$V_{EAS_{stall}} = \sqrt{\frac{W}{S}\frac{2}{\rho_0}\frac{1}{C_{L_{max}}}}$$

This is indication of the minimum equivalent airspeed you can fly with. This expression is independent of altitude.

In general, when you know the equivalent airspeed, you can use that to either convert it to true airspeed using

$$\frac{V_{Eq}}{V_{true}} = \sqrt{\frac{\rho(h)}{\rho_0}}$$

Or you can calculate the lift using the sea-level density:

$$L = \frac{1}{2}\rho_0 V_{EAS}^2 C_L = \frac{1}{2}\rho_h V_{TAS}^2 C_L$$

Usually, the sea level pressure: is not always the case due to weather conditions. This can be quite complicated if the pressure varies constantly during a flight. The flight-level is the altitude when considering the fixed sea-level pressure of 101325 Pa. This is not the true altitude, but is independent of the change in the local atmospheric pressure. Notation: FL250 = 25,000 feet above sea level, calculated using ISA. Pilots usually use Equivalent airspeed and flight-level as indication.

7. Aerospace Structures

If we analyse the body of a horse, the skin, nerves, the arteries the muscles etc, can be distinguished. The skeleton should provide strength and stiffness (resistance to deformation) to its body. This skeleton could be regarded as the ‘structure’ of the horse.

An aircraft or spacecraft also has a ‘skeleton’; the fuselage: a structure that provides strength and stiffness, protection. When removing all systems like electric wiring, isolation, hydraulics, etc. from the aircraft, the remainder is its structure.

The structure of the fuselage of an aircraft.

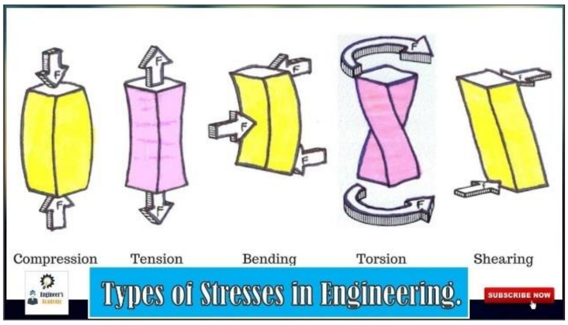
Functions of a structure

- Carrying the loads: it should take care of the loads. An aircraft is loaded by many forces and moments (bending and torsion) resulting from its operation. Loads caused by weight, temperature etc., the structure should take care of these loads and the structure should make sure that the loads are in equilibrium.
- Protective function: it should protect the payload (passengers and/or cargo) against the environment and impacts.
- Framework for attachment: All systems like electric systems, control systems, actuators, high-lift devices should be fixed and attached to the structure which keep the systems in place and support them.

Requirements for a structure:

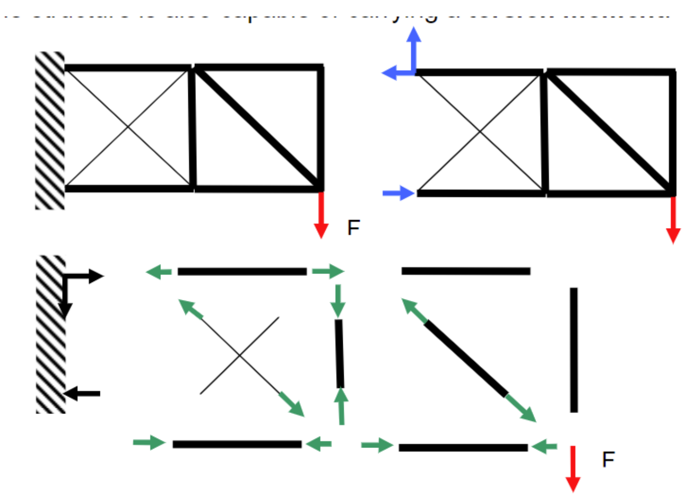
- The structure should be strong enough. It should carry the loads (like force and moments) that the structure will endure. The strength of the structure depends on the mission of the aircraft.
- The structure should have enough stiffness: A large force tends to cause the wing to bend or deflect. Stiffeners resist this deformation. Usually, the stiffness should neither be too high nor too low.
- The structure should be a lightweight structure. The empty weight of an aircraft should be as low as possible so as to fly longer and take more payloads.
- The operational life of an aircraft is durable, i.e. it should operate at least 20-30 years, hence resistance towards corrosion, fatigue.
- The structure should be cheap and cost effective.
- The availability of the aircraft or ease of its maintenance; any damage or malfunction to an aircraft should be replaces quickly.
- Limit load = Ultimate load x safety factor
- Limit load is highest expected load, ultimate load is load when failing

Structural concepts



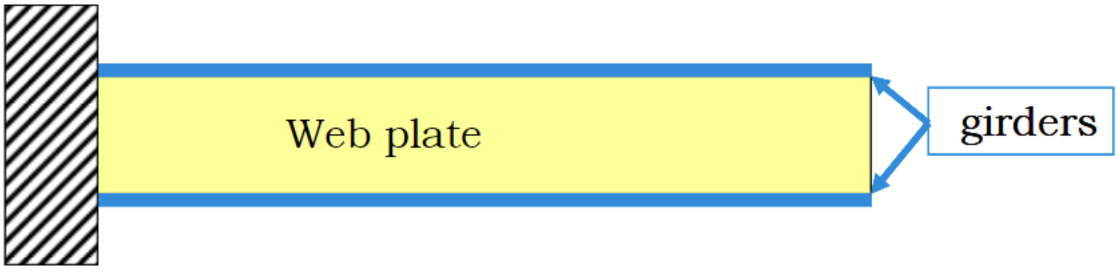
The first types of structures was the truss structures, in which the members are made by tubes, bars and wires. The tubes and wires carried all the loads; the skin had no structural significance.

A 2D truss structure is capable of carrying tensile and compressive forces as well as a bending moment. When it is extended to 3D, the forces perpendicular to the plane, or torsional moment can also be carried. Here we see the external force, the reaction forces of the wall and the members.

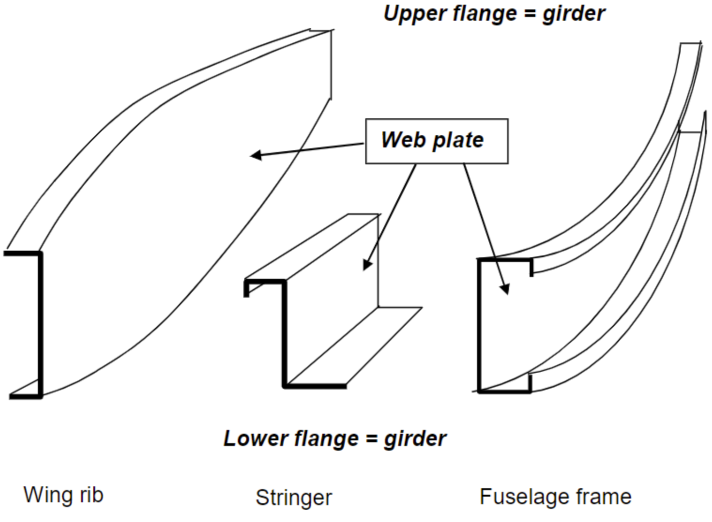


Later, metal structures were developed, using metal sheets known as **shell structures.** The supporting structure is made of beamlike elements like stringers, ribs, frames, spars etc.

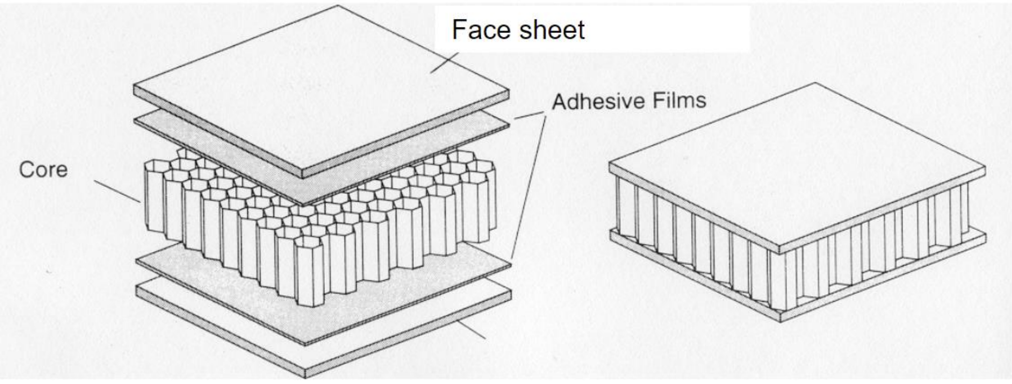
The **concept of a beam** can be derived from the truss structure if we replace the diagonal wires or tubes by a metal sheet, **web plate.** These carry the shear loading and the **girders** take care of the bending moment.



A lot elements are based on this concept:



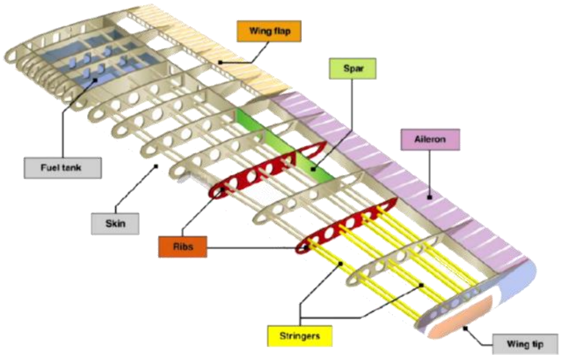
Today, a lot of structures are still made as shell structures, either made of metal alloys or composite materials. However, composite structures can also be made from sandwiches, which is beamlike. Main differences are that sandwich acts as a beam in two orthogonal directions, whereas a beam has a much higher length relative to its width. The face sheet carry the bending loads: one sheet for the tension the other for the compression. The core material should carry the shear loads and support the face sheets to prevent the buckling (in case of compression). The face sheets are bonded to the core material by adhesive films.



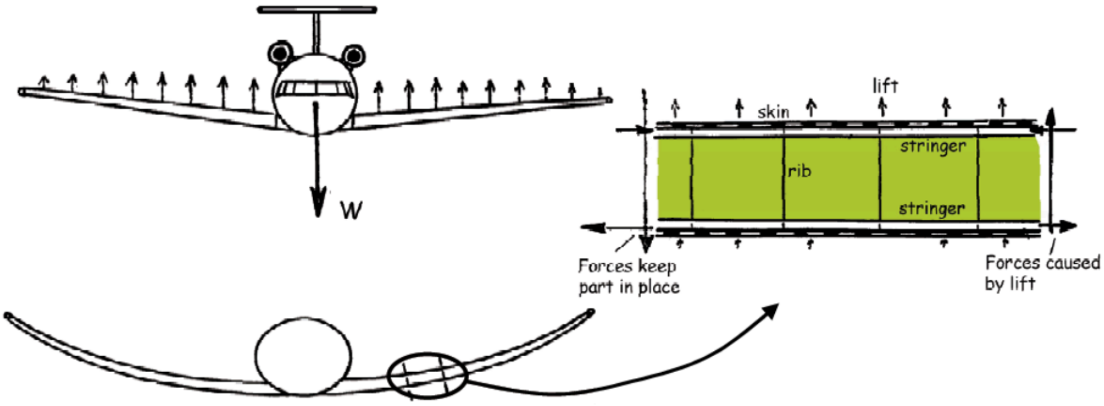
An aircraft fuselage is pressurized every flight. In this case, fatigue is a real issue for the design of a fuselage. Fatigue is a dynamic loading: the structure is loaded and unloaded many times until failure occurs at a level much lower than the failure strength of the material.

8. Aerospace Materials

Lists of beam-like structural elements in a wing



- Ribs maintain the aerodynamic profile of the wing, transfer the aerodynamic and fuel loads acting on the skin to the rest of the wing structure, provide stability against buckling.
  Ribs are placed perpendicular to the flight direction so as to minimize the weight.
- Spars are not part of the aerodynamic shape, but only carry the wing bending loads. They are I-beams. The girders carry the normal forces and web plates carry the shear forces.
- Stringers are used to transfer the stresses and loads from the skin to the formers.



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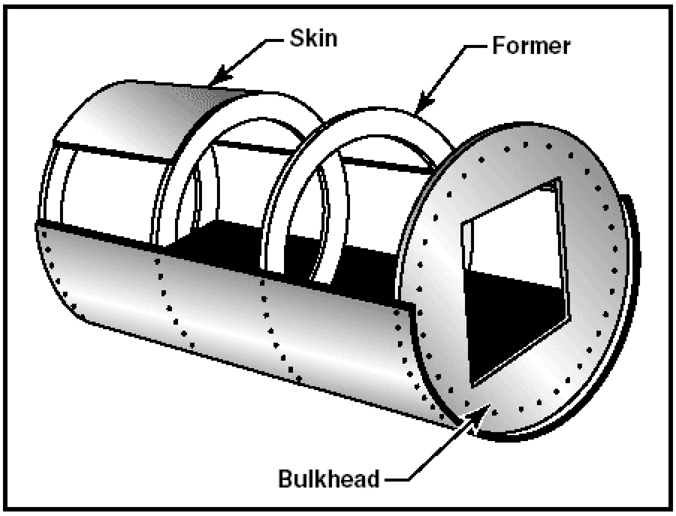
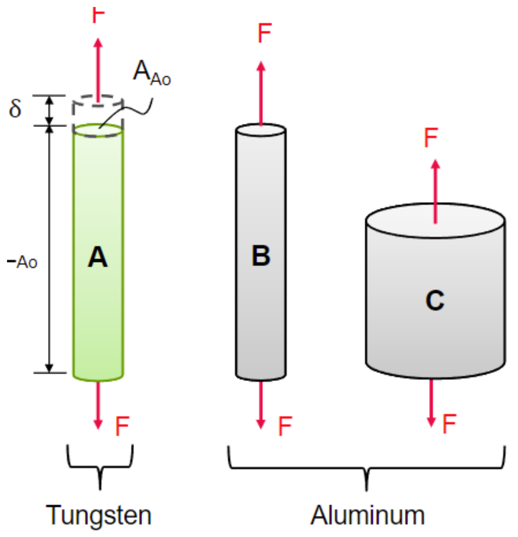
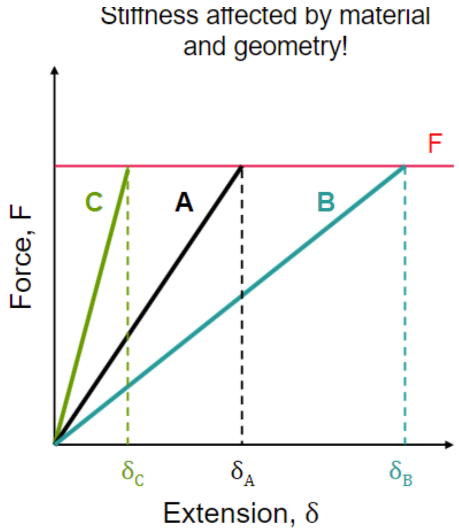


If a tension force is applied on a surface area (see figure), then the material is going to elongate by an extension δ. The stress σ is the force applied per unit area, and the strain ε (rek) says how much the material is extended. How much extension you get for a certain amount of force depends on the geometry of the material and the type of material (C is the easiest because of light material, B is harder because of heavier material, C is the hardest because of hard material and wide geometry).





So,

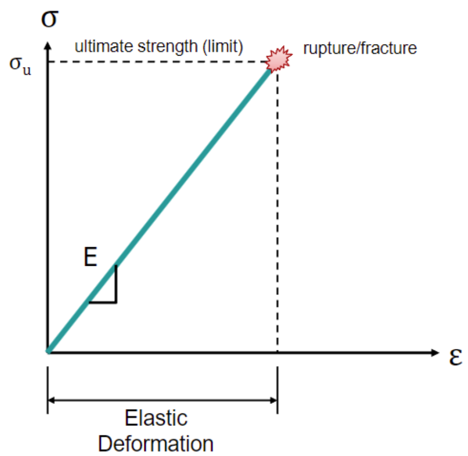
$$\sigma = \frac{F}{A_0}, \varepsilon = \frac{\Delta L}{L_0}$$

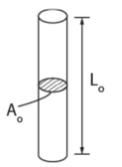
Where A_0 and L_0 are initial conditions before the stress.

The stress-strain curve tells you the relationship between how much strain you get for a certain amount of stress. For brittle materials (easy to break, not flexible), there is only a linear elastic region, where the gradient is the material property Young's Modulus:

$$E = \frac{\sigma}{\varepsilon}$$

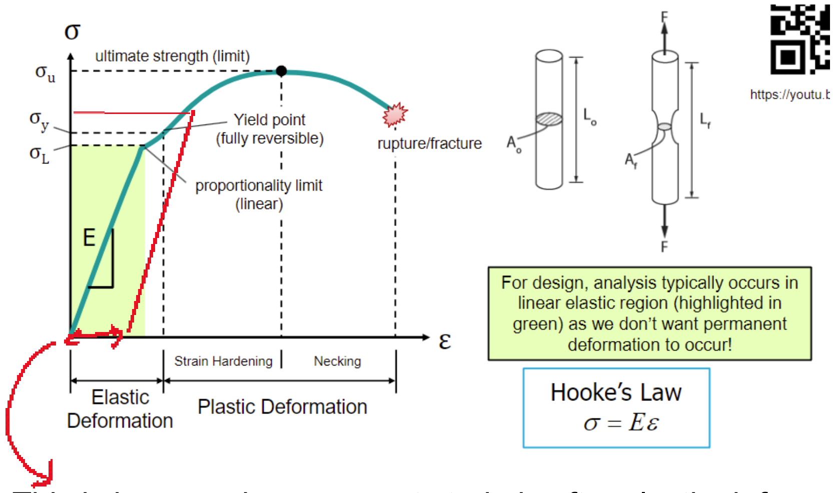
Elastic means that after the stress is applied, the material is going back to its original state. For ductile materials (more flexible), there is an elastic region, and a plastic deformation region.





Hooke's Law
$\sigma = E\varepsilon$

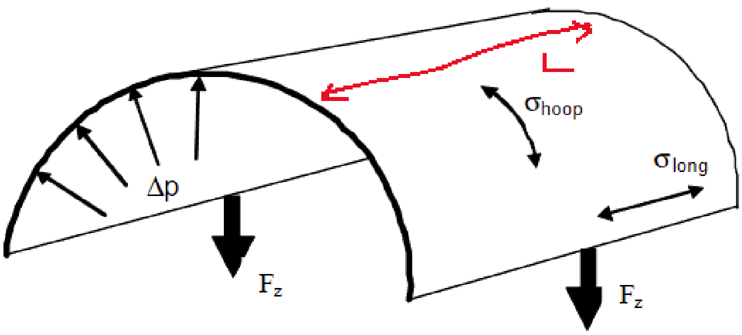
For design, typically keep strains below 30% of ultimate strength



This is how much permanent strain is after elastic deformation is passed.

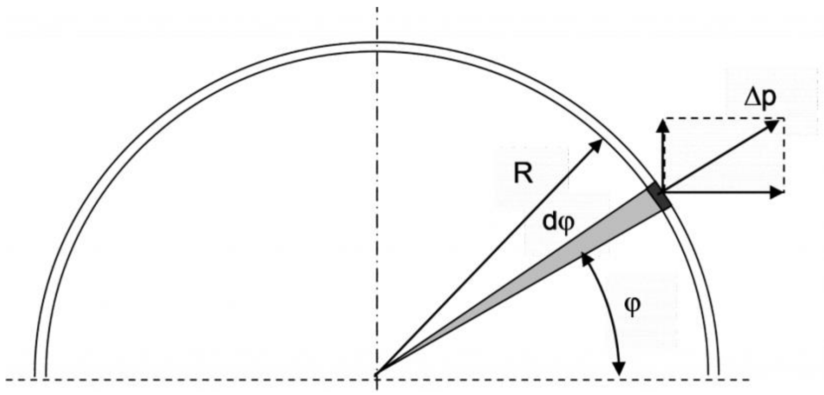
Plastic deformation, thus, is a permanent change. After the ultimate strength point in the figure, the stress decreases as strain increases, and there is finally a fracture. The strain hardening is the section of plastic deformation where there is still increase in stress (hardening), and necking is when it's reversed.

Since the inside of the fuselage is pressurized, the material causes stresses on the inside of the fuselage. $\Delta p = p_{insi} - p_{out}$. Consider the upper half of the fuselage, a semicircular vessel.



The σ_{hoop} is the circumferential stress and causes deformation in the y direction. σ_{long} is the stress in the x/z direction (elongation). Since we cut out a section, there are reaction hoop forces F_z caused by stresses. Stresses, are, in fact, reaction forces by the material as a result of the pressure acting on it.

To calculate the σ_{hoop}, consider the following diagram.



We consider the black arc to be infinitesimal small, with $L_{arc} = \theta r = R\ d\varphi$, applying equilibrium in vertical direction

$$\sum_i F_x \rightarrow +: 2F_z - F_{due\ to\ pressure} = 0$$

Since $F_{due\ to\ pressure}$ is only for the vertical direction, and pressure is defined as perpendicular to the semicircle, we need to calculate the vertical component, and integrate it over the whole angle of 180 degrees,

$$F_{due\ to\ \ pressure} = \int_0^{\pi} (\Delta p \sin(\varphi)) R\ L\ d\varphi = \Delta pR\ L \int_0^{\pi} \sin\ (\varphi)\ d\varphi = 2\Delta pRL$$

Where L is the longitudinal length of the vessel. So, $R\ L\ d\varphi$ represents an area of length L and infinitesimal width $R\ d\varphi$.

Let t be the thickness of the vessel, then

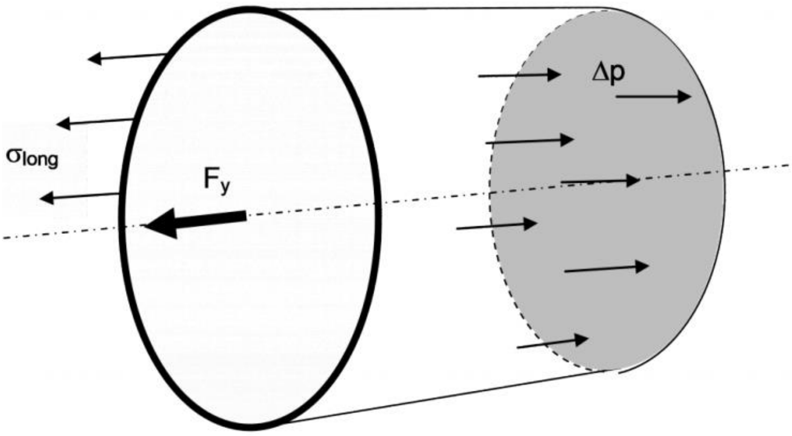
$$F_z = \sigma_{hoop} Lt$$

Equilibrium,

$$2\sigma_{hoop} Lt = 2\Delta pRL \rightarrow \sigma_{hoop} = \frac{\Delta pR}{t}$$

This is the stress that tries to elongate it in the y direction.

For the longitudinal stress, consider



F_y is the resultant force to the inertial pressure, hence it can be expressed as:

$$\sum_i F_y : 0 = F_{due\ to\ pressure} - F_{stress\ by\ the\ material}$$

This is the force exerting on the whole surface. Since the force exerted by pressure is equal to the stresses on the surface. The forces due to pressure is the pressure difference multiplied by the cross-sectional area:

$$F_{due\ to\ pressure} = \Delta p \pi R^2$$

The stress of the material is due to the pressure is the force acting on the thin-walled section

$$F_y = \sigma_{long} 2\pi R t$$

Equating,

$$\sigma_{long} 2\pi R t = \Delta p \pi R^2 \rightarrow \sigma_{long} = \frac{\Delta p R}{2t}$$

When calculating the minimum thickness of a pressurized vessel, always calculate using both the hoop and longitudinal stress. After which you choose the largest value for the minimum thickness.

Example:

A load from point A to point B is transferred on a transfer bar either from Steel or Aluminum. To determine which offers the lightest the solution, with the following data:

Tensile load = 1000 kN

Length of the bar = 2m

Yield strength steel: 550 N/mm^2

Density steel: 7.8 kg/dm^3

Yield strength aluminum: 280 N/mm^2

Density aluminum: 2.8 kg/dm^3

Solution:

The maximum stress of steel and aluminum are: , with $L = 2$ and $F = 1000\ kN$. We want to know the required weight in order to transfer this load knowing that $m = \rho A L$.

$$A_{steel} = \frac{F}{\sigma_{steel}} = \frac{1000}{550} = 1818\ mm^2$$

$$A_{aluminum} = \frac{F}{\sigma_{aluminum}} = \frac{1000}{280} = 3571\ mm^2$$

$$m_{steel} = 20 \cdot 0.1818 \cdot 7.8 = 28.4\ kg$$

$$m_{aluminum} = 20 \cdot 0.3571 \cdot 2.8 = 20.0\ kg$$

This gives that aluminum gives the lightest solution.

Aerospace Materials

A material is defined as a ‘substance, constituent, or element used to build parts, products, or structures.’

Material properties, such as mechanical, electrical, and physical characteristics, depend on its composition, not its shape. Common materials include metals like steel and aluminum, wood, ceramics, and polymers. In the aerospace industry, lightweight materials with a high performance-to-weight ratio are essential, offering high specific strength and stiffness. Examples of such materials include aluminum alloys, titanium alloys, carbon fiber composites, and glass fiber composites, all known for their low densities.

Material groups:

- Lightweight metal alloys (aluminum, steel). An alloy (legering) is made by adding alloying element to the purified metal in order to increase the properties of the pure metal. This often increases the yield strength and ultimate strength with a factor of 4-7.
- Composite materials. Composites are materials composed of different constituents (individual chemical elements), e.g., fibers (vezels) and a polymer. Fibers provide the strength and stiffness to the material; the polymer provides the support of the fiber, takes care of the load transfer between fibers, and protects the fibers.

Requirements for a material:

Many requirements for a structure also comply with that of a material. BUT, the difference between the requirements are that structural requirements are satisfied by changing the **geometry of the structure.** Adding strength to a material can be done by alloying new elements or changing the state of the material:

- The manufacturability or workshop properties of the material. It is important for the manufacturer that processes like forming and machining should be easy for the materials (e.g. aluminum is easy for manufacturability, whereas titanium alloys aren’t)
- Physical properties like the electrical conductivity and the thermal expansion (uitzetting) are important for specific features. Aluminum alloys are good materials for a cage of Faraday, for example. For composites this is more difficult: sometimes extra strips are required for protection against lightning.
- Usually, the ultimate strength of a material is not the same of the structure, because the structure needs to carry other loads.

From materials to manufacturing:

- Materials are the substances used to fabricate structures, done by retrieving from resources like ores and oil. Once retrieved, they are processed into sheets, plates etc. by machining, casting etc.
- Once the materials are created, the elements are assembled into structures. The properties of a structure are directly related to the material properties although not identical: structural properties are influenced heavily by the shape/design.. Furthermore, not every shape can be made of any material, because the materials have different physical properties.
- Manufacturing process is also dependent on the material. Metals can be melted, so welding is an available production process for metals. However, fibers reinforced polymers cannot be welded.
- Lastly, the shape of the structure also determines the manufacturing process. If we want to fabricate a sheet metal wing rib we may use a forming process. But replacing that with a machined one will change the shape of the wing rib.

In short, there is a strong interrelationship between the three entities 'material', 'structure or shape', and 'manufacturing process'.

Summary of material properties:

- Metals: ductile, isotropic
- Ceramics: too brittle, strong, often used as heat protectant, thermal insulation
- Polymers: weak and low stiffness (very flexible), often used in composites
- Composites: combination of layers of different materials. They produce structural or functional properties not present in the individual components, strong and anisotropic. They have strength in different directions due to layers. The maximum strength of one composite, is the strength of the layers, when taking the material volume into account. In transverse strength, its the strength of the resin. Resin: protective, fibre: stiffness and strength
- • **Fail-Safe** focuses on how a system behaves after a failure happens, ensuring that the failure doesn't lead to a catastrophic event.
- • **Safe-Life** focuses on preventing failures by retiring the component before it has a chance to fail.

c) Calculate the maximum strength of the composite plate in 0-degree direction (2 points).

Answer:

$$\sigma_{\text{max0 degrees}} = 75\%\ \sigma_{\text{max}_{\text{Fibre dir}}} + 25\%\ \sigma_{\text{max}_{\text{transversal dir}}}$$

$$= 0.75 \cdot (\text{answer a}) + 0.25 \cdot (\text{answer b})$$

$$= 0.75 \cdot 1065 + 0.25 \cdot 50 = 811.25 \text{ MPa}$$

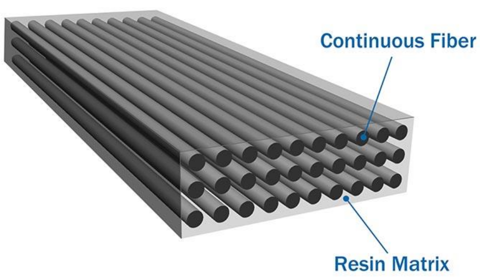
d) Calculate the maximum strength of the composite plate in 90-degree direction (2 points).

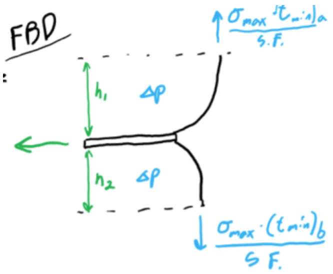
Answer:

$$\sigma_{\text{max90 degrees}} = 25\%\ \sigma_{\text{max}_{\text{Fibre dir}}} + 75\%\ \sigma_{\text{max}_{\text{transversal dir}}}$$

$$= 0.25 \cdot (\text{answer a}) + 0.75 \cdot (\text{answer b})$$

$$= 0.25 \cdot 1065 + 0.75 \cdot 50 = 303.75 \text{ MPa}$$





$$N - \Delta p (h_1 + h_2) = 0$$

$$\Rightarrow N = \Delta p (h_1 + h_2)$$

$$= 60 \times 10^3 \text{ Pa} (1 \text{ m} + 0.25 \text{ m})$$

Lecture 10: Flight Mechanics

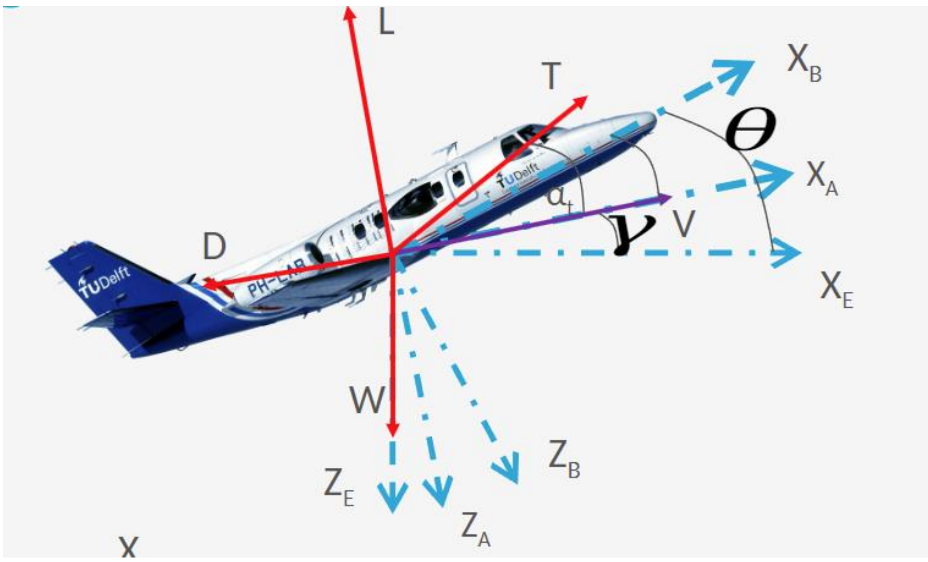
Newton's laws only apply to inertial reference frames. In aerospace, we usually simplify the earth to be flat (inertial), however, in reality, since the earth is round, the airplane flying makes a centripetal motion, where $a_{cent} = \frac{V^2}{R_E}$. This is the acceleration to make a circular path. This acceleration, however, is relatively small and can be neglected.

Reference frames:

- Body axis system x_B and z_B, parallel to the nose of the aircraft. Inclined at an angle θ.
- Moving earth axis system x_E and z_E, parallel to the flat ground, but moving with the aircraft. This is where the weight is defined.
- Air-path reference frame x_B This is where the airspeed, drag and lift are defined. The angle with the horizontal is γ. So, $\theta = \gamma + \alpha$.

The velocity is not usually in the direction of the nose. The thrust is also not necessarily in the direction of the airspeed vector, thus making an thrust angle of attack α_T. The actual angle of attack, α, is the angle between the velocity and the body x axis.

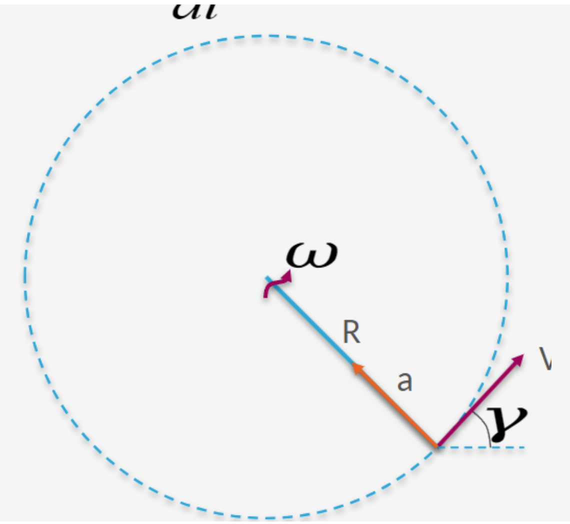
FBD:

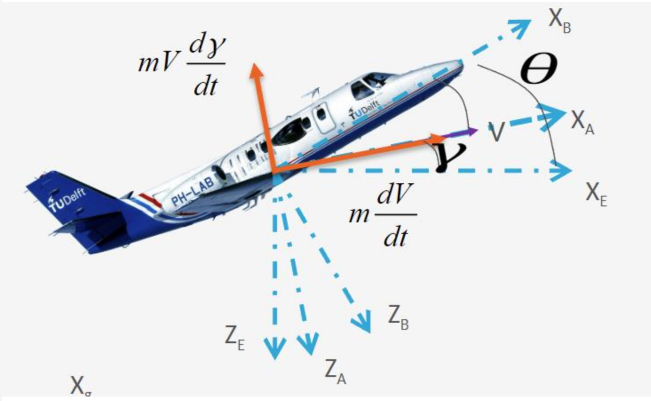


Kinetic diagram is where the accelerations and velocity is defined. There is an acceleration in the direction of velocity, and a centripetal acceleration perpendicular to the velocity, causing rotational motion.

$$a_{\parallel} = \frac{dV}{dt}, a_{\perp} = \frac{V^2}{R}$$

Since $V = \frac{d\gamma}{dt}R$, $a_{\parallel} = \frac{dV}{dt}$, $a_{\perp} = V\,\frac{d\gamma}{dt}$



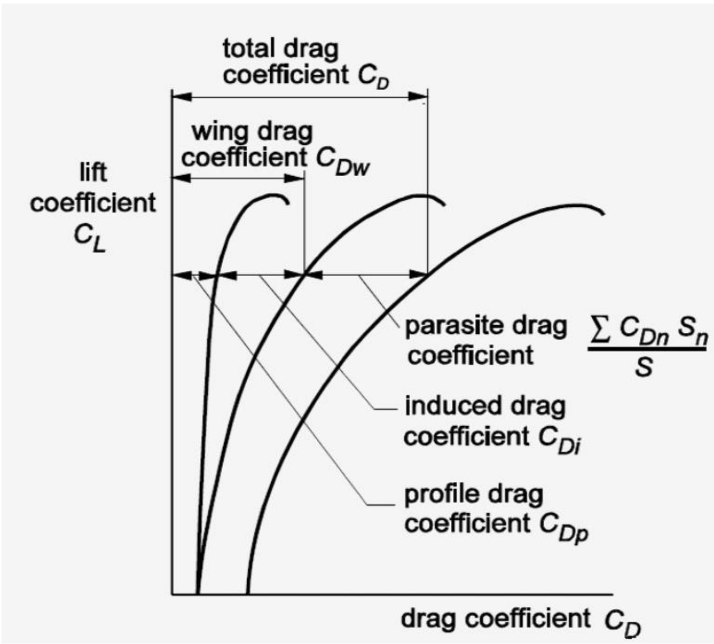


Equations of motion:

$$\sum F_{\perp}: L + T\sin(\alpha_T) - W\cos\gamma = \frac{W}{g}V\frac{d\gamma}{dt}$$

$$\sum F_{\parallel}: T\cos(\alpha_T) - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

The independent variable is time. The state variables (defines the state of the airplane) are V and γ. The remaining variables can be expressed as functions of the state variables.



Total drag consists of parasitic drag and lift-induced drag,

$$C_D = C_{D0} + \frac{C_L^2}{\pi A e}$$

However, if you plot C_D against C_L^2, in reality, it is not entirely a straight line. To be more precise (and hence not waste any fuel), the parabolic lift drag polar is in reality,

$$C_D = C_{D0} + k_1 C_L + k_2 C_L^2$$

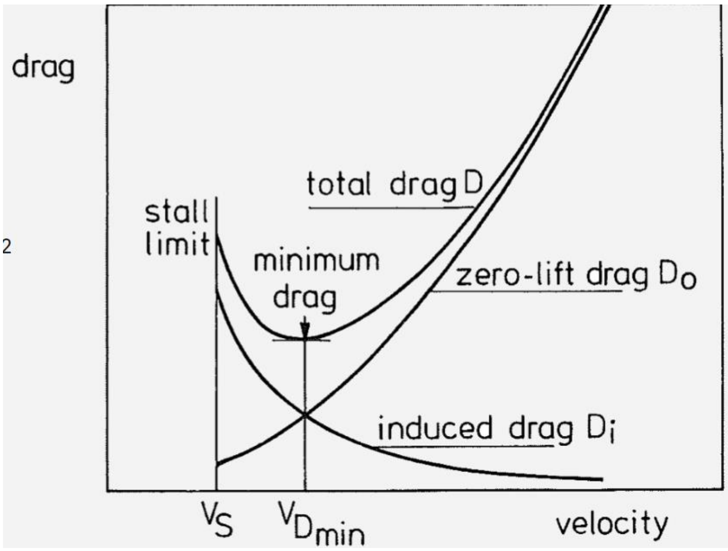
In steady flight,

$$C_L = \frac{2W}{\rho V^2 S}$$

$$D = \left(C_{D0} + k_1 \frac{2W}{\rho V^2 S} + k_2 \left(\frac{2W}{\rho V^2 S} \right)^2 \right) \frac{1}{2} \rho V^2 S$$

$$D = C_{D0} \frac{1}{2} \rho V^2 S + k_1 W + k_2 \frac{2W^2}{\rho V^2 S} = f(V^2) + f\left(\frac{1}{V}\right)^2$$

At higher airspeeds, $f\left(\frac{1}{V^2}\right) \to 0$, meaning the zero lift drag is of more influence. This means that military aircraft need a low area S. At lower airspeeds, $f(V^2)$ is of more influence. Thus, at lower airspeeds, you need higher aspect ratios.



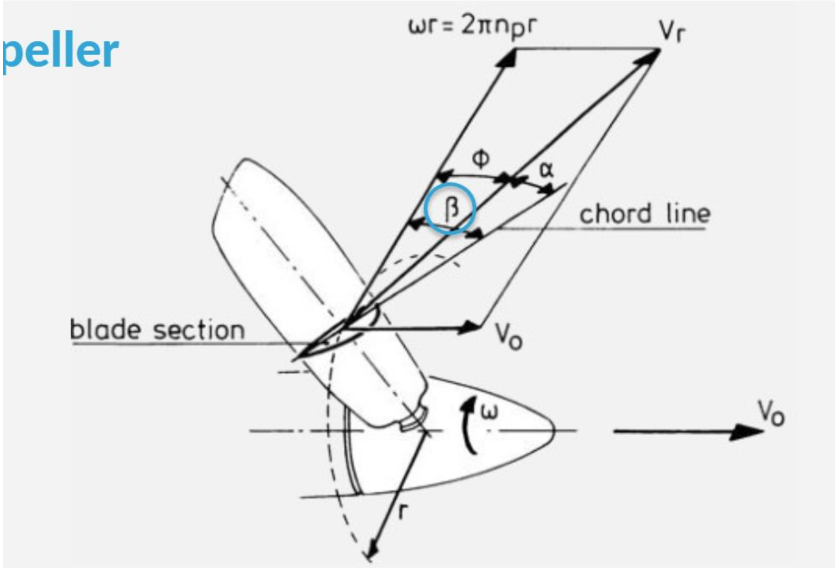
In a jet engine, $T = \dot{m}(V_j - V_0)$.

The jet velocity mainly depends on the throttle setting and compression rate.

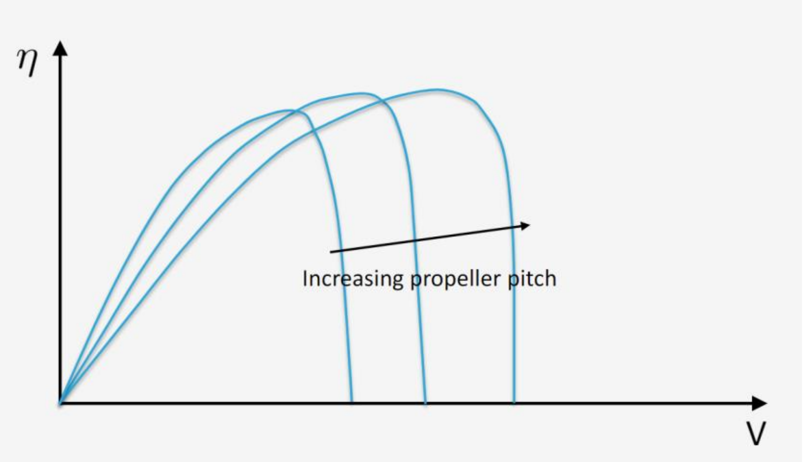
If we increase the airspeed, the jet velocity stays constant, but the mass flow increases as a higher velocity results in an increase of air entering the engine

($\dot{m} = \rho A V_0$). As a result, the thrust generation by a jet engine can be considered more or less constant as a function of airspeed.

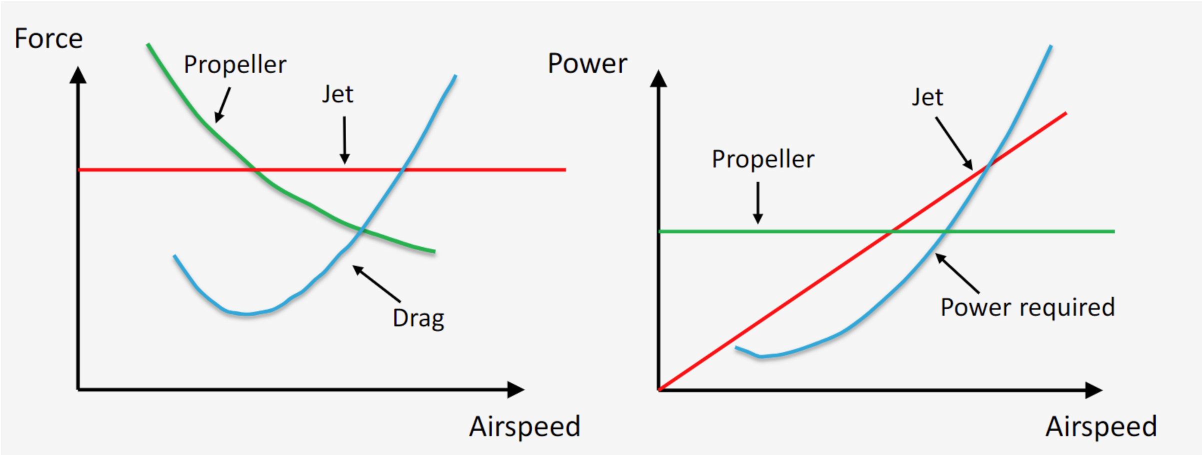
The blade section of a propeller generates lift and drag.



The blade of a propeller can be considered an airfoil which creates an aerodynamic force. The velocity vector, $\overrightarrow{v_R}$, is the resultant of the angular velocity (rotational velocity of the blade) and the airspeed → $\overrightarrow{v_R} = \overrightarrow{v_{blade}} + \overrightarrow{v_0}$. The angle of attack, in this case is the angle between v_R and the camber of the airfoil. The aerodynamic force generated perpendicular to the blade, is the thrust. β is the pitch angle, angle between camber and vertical. The pitch angle can be changed by the varied, resulting in different local angle of attacks. This changes the lift coefficient and therefore the thrust can be controlled. Thus, for every amirspeed, the optimum pitch angle can be selected.



The propulsive efficiency is, $\eta_{prop} = \frac{P_a}{P_{br}} = \frac{TV_0}{P_{br}}$ is constant. If the shaft power is also constant, then the power available is also constant. Therefore, $T \propto \frac{1}{V}$. The performance diagrams can be summarized:



Lecture 11: Horizontal flight

At steady horizontal flight, there is no acceleration as well as a flight-angle, thus $\sin\gamma = 0, \cos(\gamma) = 1, \frac{dV}{dt} = 0, \frac{d\gamma}{dt} = 0$. These conditions will hugely simplify our equations of motion:

$$\sum F_{\perp}: L + T\sin(\alpha_T) - W\cos\gamma = \frac{W}{g}V\frac{d\gamma}{dt}$$

$$L = W$$

$$\sum F_{\parallel}: T\cos(\alpha_T) - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$T = D$$

A performance diagram relates the power required ($P = DV$ and power available $P = TV$) to the airspeed. For a propeller engine, P_a is constant.

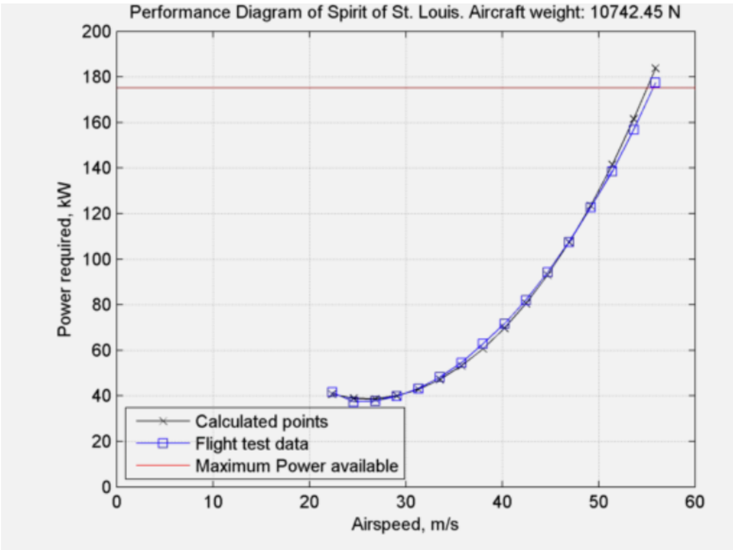
In order to determine the points on the diagram (P, V), there are three steps:

- If C_L is given, calculate the airspeed using: $V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$, if V is given, then do the reverse.
- Calculate the C_D using: $C_D = C_{D_0} + k_1 C_L + k_2 C_L^2$.
- Calculate the power required

To express power required as a function of C_D and C_L, a trick can be applied:

$$P_r = D\frac{L}{L}V = \frac{D}{L}WV = \frac{C_D}{C_L}WV$$

This way, the performance diagram can be sketched. Power available



Minimum Airspeed

The minimum airspeed, also known as the stall speed, is the minimum velocity at which aircraft can still fly steady horizontally. When having a performance diagram, just search for the smallest value.

Recall the airspeed equation,

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

When $C_L = max$, the airspeed is minimum. So the $C_{L_{max}}$, which is a given parameter, determines the minimum airspeed.

This equation is the same for a jet aircraft.

Maximum airspeed

The maximum airspeed on the performance diagram is the intersection between the maximum power available and power required, or, in other words, the airspeed in which $T = V$. This is, essentially the point at which $D = T$ ($P_r = P_{a_{max}} \rightarrow DV = TV$).

$$P_a = P_r = \frac{C_D}{C_L}WV$$

$$P_{a_{max}} = \frac{C_D}{C_L}WV = \frac{C_{D_0} + k_1 C_L + k_2 C_L^2}{C_L} W \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

After some manipulation,

$$\frac{(C_{D_0} + \mathrm{k}C_L^2)^2}{C_L^3} = P_a^2 \frac{S}{W^3}\frac{\rho}{2}$$

In this equation, power available as well as all the other variables are constant. You can solve for C_L and put it into the airspeed equation which gives you maximum speed.

For a jet aircraft, power available is also a function of airspeed. Which is why the maximum airspeed have to be calculated differently.

We know the thrust is independent of airspeed and equal to the drag. So, the maximum thrust, or just the thrust as there is no maximum as function of airspeed, can be done as follows

$$T_{max} = D$$

$$T_{max} = D\frac{L}{L} \qquad \textit{We multiply by 1}$$

$$T_{max} = \frac{C_D}{C_L}W \qquad \frac{D}{L} = \frac{C_D}{C_L} \textit{ and } L = W \textit{ from Equation 2}$$

$$T_{max} = \frac{C_{D_0} + k_1 C_L + k_2 C_L^2}{C_L} \cdot W \qquad \textit{From the lift-drag polar we know } C_D = C_{D_0} + k_1 C_L + k_2 C_L^2$$

This final equation can be rewritten as a parabolic relation as follows:

$$T_{max} = \frac{C_{D_0} + k_1 C_L + k_2 C_L^2}{C_L} \cdot W$$

$$\frac{T_{max}}{W} = \frac{C_{D_0} + k_1 C_L + k_2 C_L^2}{C_L} \qquad \textit{Divide by } W$$

$$\frac{T_{max}}{W} \cdot C_L = C_{D_0} + k_1 C_L + k_2 C_L^2 \qquad \textit{Multiply with } C_L$$

$$k_2 C_L^2 + \left(k_1 - \frac{T_{max}}{W}\right) C_L + C_{D_0} = 0 \qquad \textit{Group all terms with } C_L^2,\ C_L \textit{ and } C_{D_0}$$

This quadratic equation can be solved for C_L by means of the quadratic formula:

$$C_{L_{\mathrm{opt}}} = \frac{-\left(k_1 - \frac{T_{max}}{W}\right) \pm \sqrt{\left(k_1 - \frac{T_{max}}{W}\right)^2 - 4k_2 C_{D_0}}}{2k_2} \tag{4}$$

This C_L can also be added into the airspeed equation.

Keep in mind that once solving equations, you may get more than solution, always choose the smallest of them.

Range

In order to have maximum specific range, we want to use a minimum amount of fuel per unit of distance. This means that the specific range $\frac{V}{F} = \frac{V}{\dot{m}_f}$ (speed over fuel flow) should be minimised. In this derivation we assume steady straight and symmetric flight, such that the previous equations hold.

$$F = c_p P_{br} = \frac{c_p}{\eta_{prop}} P_a = \frac{c_p}{\eta_{prop}} P_r = \frac{c_p}{\eta_{prop}} DV$$

Thus,

$$\frac{V}{F} = \frac{V}{\frac{c_p}{\eta_{prop}} DV} = \frac{\eta_{prop}}{c_p} \frac{1}{D}$$

All are constant except for the drag. This means that for maximum range, we need to minimize drag.

$$D = \frac{C_D}{C_L} W$$

Thus, $\left(\frac{C_D}{C_L}\right)_{min} \rightarrow \left(\frac{C_L}{C_D}\right)_{max}$

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = 0$$

$$\frac{d}{dC_L}\left(\frac{C_{D_0}}{C_L} + k_1 + k_2 C_L\right) = 0$$

$$-\frac{C_{D_0}}{C_L^2} + k_2 = 0 \rightarrow C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{k_2}}$$

This value can be used to determine the optimal airspeed.

As for a jet aircraft, the efficiency is not constant, so we have to use the thrust.

We can relate fuel flow F by the generated thrust using

$$F = c_p T = c_p D$$

Where c_P is a constant.

Thus,

$$\frac{V_0}{F} = \frac{V}{c_p D}$$

If we want to maximise V/F, we need to maximise V/D (since $c_T = \text{const}$). Furthermore, $(V/D)_{\text{max}}$ is the same as $(D/V)_{\text{min}}$. This expression can be used in order to find the optimal aerodynamic condition as follows:

$$\left(\frac{D}{V}\right)_{min} = \left(\frac{D}{L} \cdot L \cdot \frac{1}{V}\right)_{min} \qquad \textit{We multiply with } \frac{L}{L} = 1$$

$$\left(\frac{D}{V}\right)_{min} = \left(\frac{C_D}{C_L} \cdot W \cdot \frac{1}{V}\right)_{min} \qquad \textit{Since } \frac{D}{L} = \frac{C_D}{C_L} \textit{ and } L = W$$

$$\left(\frac{D}{V}\right)_{min} = \left(\frac{C_D}{C_L} \cdot W \cdot \frac{1}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}}\right)_{min} \qquad \textit{From Equation 1}$$

$$\left(\frac{D}{V}\right)_{min} = \left(\frac{W}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_L}{C_D^2}}}\right)_{min} \qquad \textit{We put everything under the square root}$$

$$\left(\frac{D}{V}\right)_{min} \rightarrow \left(\frac{C_L}{C_D^2}\right)_{max}$$

Now we know at what aerodynamic condition maximum specific range occurs, we need to find at what lift coefficient this is. For this we differentiate the expression just found with respect to C_L, and put it equal to 0:

$$\frac{\mathrm{d}}{\mathrm{d}C_L}\left(\frac{C_L}{C_D^2}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}C_L}\left(\frac{C_L}{C_D^2}\right) = \frac{C_D^2 \cdot 1 - C_L \cdot 2C_D\frac{\mathrm{d}C_D}{\mathrm{d}C_L}}{C_D^4} = 0 \qquad \textit{Using the quotient rule}$$

$$\frac{C_D^2 \cdot 1 - C_L \cdot 2C_D\frac{\mathrm{d}C_D}{\mathrm{d}C_L}}{C_D^4} = 0 \rightarrow \frac{\mathrm{d}C_D}{\mathrm{d}C_L} = \frac{1}{2}\frac{C_D}{C_L}$$

Endurance

In order to have maximum endurance, the fuel flow needs to be minimized.

$$F = c_p P_{br} = \frac{c_p}{\eta_{prop}} P_a = \frac{c_p}{\eta_{prop}} P_r = \frac{c_p}{\eta_{prop}} DV$$

All are constant, except DV, thus we need to minimize DV

$$(DV)_{min} = \frac{C_D}{C_L} W \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{W^3}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

All are constant except $\frac{C_D^2}{C_L^3}$, we means we have to minimize

$$\left(\frac{C_D^2}{C_L^3}\right)_{min} \rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{max}$$

Thus,

$$\frac{d}{d_{CL}}\left(\frac{C_L^3}{C_D^2}\right) = 0$$

Applying the quotient rule

$$\frac{3C_D^2 C_L^2 - 2C_L^3 C_D \dfrac{dC_D}{dC_L}}{C_D^4}$$

We applied the chain rule here: $\frac{d}{d_{C_L}}(C_D^2) = 2C_D \frac{dC_D}{dC_L}$

Substituting $\frac{dC_D}{dC_L} = -\frac{C_{D_0}}{C_L^2} + k_2$ and $C_D = C_{D_0} + k_1 C_L + k_2 C_L^2$ into the equation, we get

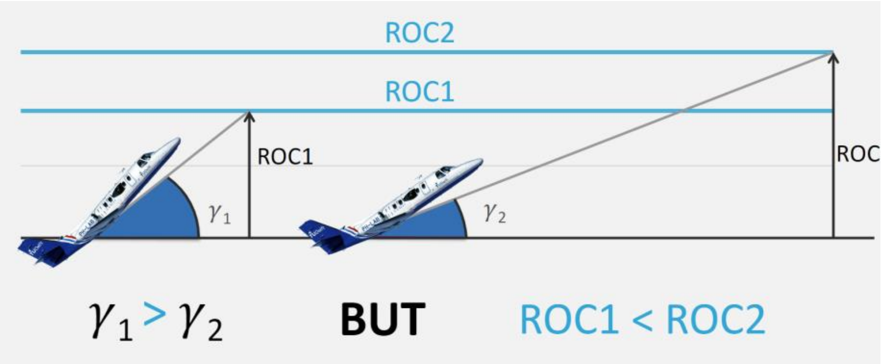
$$C_{L_{opt}} = \frac{k_1 \mp \sqrt{k_1^2 + 12k_2 C_{D_0}}}{2k_2}$$

This expression can be put into the airspeed equation and the fuel flow, to get the corresponding airspeed equation and fuel flow for maximum endurance.

Lecture 12: Climbing, Descending & Descending Flight

When we are climbing, or taking off, the flight path-angle is positive and we experience a difference in altitude.

During the climb, the flight path angle, or climb angle, indicates how steep the aircraft climbs



The rate of climb (ROC) is the vertical component of the airspeed vector. It, therefore, is the airspeed component for altitude, thus

$$ROC = V \sin\gamma$$

When γ is higher, the rate of climb decreases.

The climb angle γ is important when passing mountaints, whereas the ROC is important when gaining altitude quickly.

If we assume the climb angle to be constant, so climbing over a straight line. Thus $\frac{d\gamma}{dt} = 0$.

If we assume the airspeed, so steady flight, $\frac{dV}{dt} = 0$

When flying at small climb angles: $\cos(\gamma) \approx 1$.

The equations of motions for climbing and descending is:

$$\sum F_{\perp}: L + T\sin(\alpha_T) - W\cos\gamma = \frac{W}{g} V \frac{d\gamma}{dt}$$

$$L = W$$

$$\sum F_{\parallel}: T\cos(\alpha_T) - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\sin\gamma = \frac{T - D}{W}$$

Substituting into the ROC:

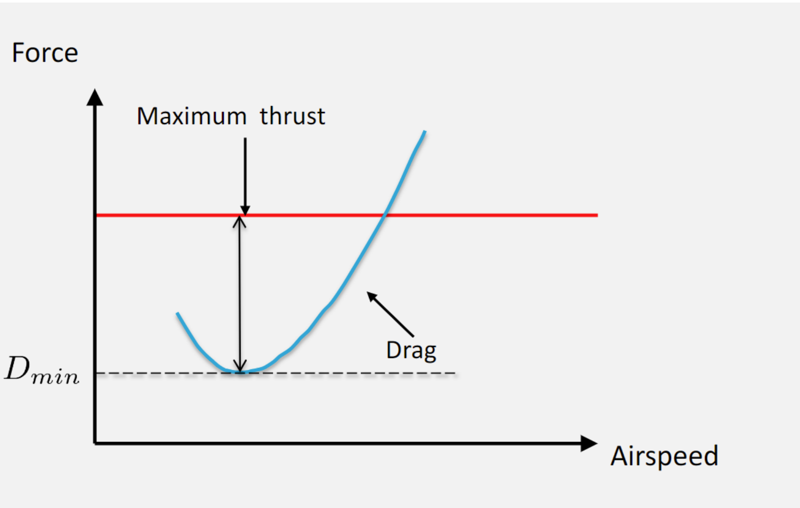
$$ROC = V\sin\gamma = \frac{TV - DV}{W} = \frac{P_a - P_r}{W}$$

Maximum climb angle

At the cockpit, there is no instrument to measure the climb angle. The climb angle, or flight-path angle is: pitch angle – angle of attack. The AoA and pitch angle varies when flying faster (if you fly faster, the lift coefficient, so the AoA can be small). So, if the pilot wants to fly at maximum climb angle, he needs to know the corresponding the airspeed at which we climb the steepest. The maximum climb angle is given by:

$$\sin\gamma_{max} = \frac{(T-D)_{max}}{W}$$

A jet aircraft has constant thrust with respect to airspeed. When having the performance diagram, the maximum climb angle is when the difference in thrust and drag is the highest.



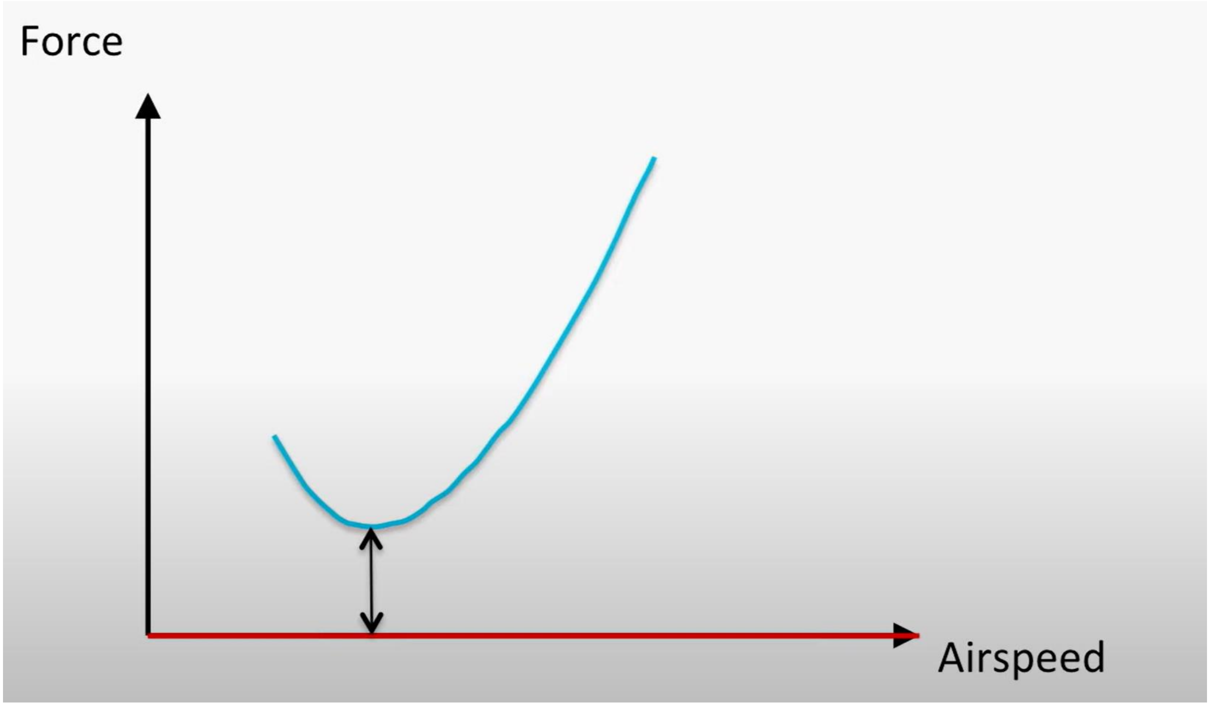
So, the maximum climb angle can be achieved when the aerodynamic drag is minimum:

$$C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{k_2}} \rightarrow V_{opt} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{\sqrt{\frac{C_{D_0}}{k_2}}}}$$

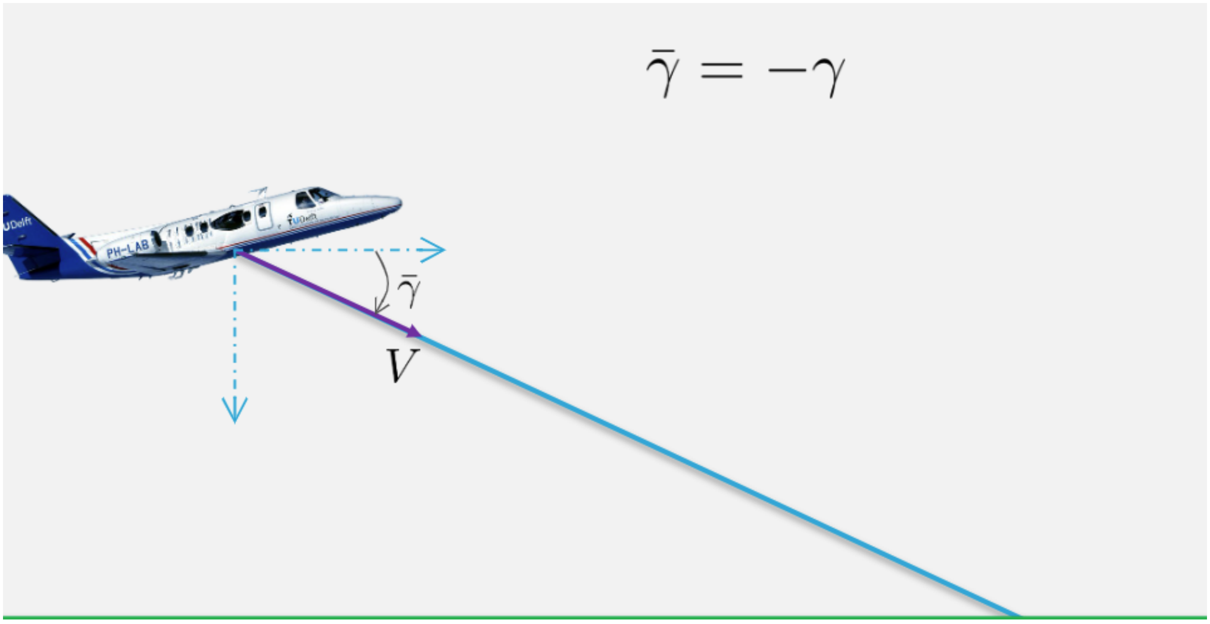
Minimum Descent Angle

A glider tries to descend as long as possible without using thrust, or an aircraft with engine troubles.

When the thrust is zero,



Here, the difference between thrust and drag is negative. If the aircraft has zero thrust, the aircraft will be performing a gliding flight, it will descend. Since the climb angle is defined positive, the climb angle will become more negative in gliding flight.



$(\tan\gamma = \frac{h}{x_{horizontal}})$

The climb angle indicates where the aircraft will hit the ground. Thus, we want to glide the furthest, $-\gamma$ needs to become the least negative, thus, the drag must be minimum. In this situation, the climb angle is given by:

$$\sin\gamma = \frac{T-D}{W} = -\frac{D}{W} = -\frac{D}{L} = -\frac{C_D}{C_L}$$

If we define $\bar{\gamma}$ as the positive version of γ, it becomes

$$\sin\bar{\gamma} = \frac{C_D}{C_L}$$

$\bar{\gamma}$ needs to minimum → $\left(\frac{C_L}{C_D}\right)_{max}$ → $C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{k_2}}$

Substituting this, the minimum climb angle $\bar{\gamma}$ equals:

$$\sin\bar{\gamma} = \frac{2C_{D_0}}{\sqrt{\frac{C_{D_0}}{k_2}}} + k_1$$

The minimum glide angle for a specific airspeed is independent of the weight, but the airspeed will be different:

$$V_{opt} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{\sqrt{\frac{C_{D_0}}{k_2}}}}$$

This means that heavier aircraft will fly faster and thus arrive at the ground faster. The location is the same, because the glide angle was an indication of the location of landing.

Maximum rate of climb in steady flight

The maximum ROC indicates the maximum vertical velocity. Therefore, it indicates how long it take to reach a certain altitude. Recall the equation for the ROC:

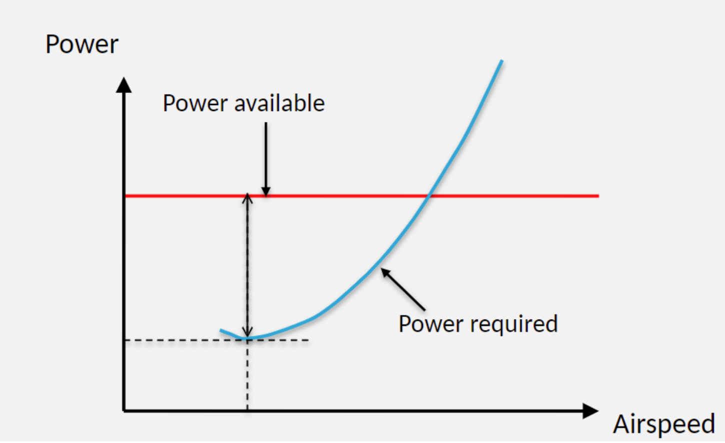
$$ROC = v\sin\gamma = \frac{P_a - P_r}{W}$$

The power available is the characteristics of the propulsion system.

The power required is the energy per second needed to overcome drag.

The maximum ROC is when the difference between the power available and power required is the largest.

In a propeller aircraft, power available is constant.



We can see that the maximum ROC is achieved when $P_r = max$. This is the same as for the maximum endurance, the optimum C_L is:

$$C_{L_{opt}} = \frac{k_1 \mp \sqrt{k_1^2 + 12k_2 C_{D_0}}}{2k_2}$$

This can be put into the airspeed equation to calculate the corresponding airspeed. You can then calculate drag to calculate power required. And when the aircraft weight is known, then the maximum power available is a given.

Minimum rate of descent

When gliding, the aircraft is descending with zero thrust, thus the ROC is negative and given by

$$ROC = \frac{-P_r}{W} = -\frac{DV}{W}$$

$$ROC = -\frac{DV}{W} = -\frac{\frac{C_D}{C_L} W \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}}{W} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

In order to get the minimum rate of descent, which is desired when gliding, the ROC needs to become as negative as possible → $\left(\frac{C_L^3}{C_D^2}\right)_{max}$

This is the same as $(P_r)_{min}$ →

$$C_{L_{opt}} = \frac{k_1 \mp \sqrt{k_1^2 + 12k_2 C_{D_0}}}{2k_2}$$

You can then substitute it into the airspeed equation and calculate drag and, in turn, power required to find the minimum rate of descent.

Summary

Horizontal, steady flight:

Most of the time, for jet aircraft, use thrust. For propeller aircraft, use power available.

Route	Propeller	Jet
Performance diagram	$T \propto \frac{1}{V}, P_a = cons,$ $\eta_{pr} = const$	$P_a \propto \frac{1}{V}, T = cons$
Minimum airspeed	$V_{st} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$	$V_{st} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$
Maximum airspeed	$P_r = P_a = \frac{C_D}{C_L} W \sqrt{\frac{2W}{\rho S C_{Lopt}}}$ $C_D = C_{D0} + k_1 C_L + k_2 C_L^2$	$T = D = \frac{C_D}{C_{Lopt}} W$ $C_D = C_{D0} + k_1 C_L + k_2 C_L^2$
Minimum drag	$\left(\frac{C_L}{C_D}\right)_{max}, D = \frac{C_D}{C_L} W$	$\left(\frac{C_L}{C_D}\right)_{max}, D = \frac{C_D}{C_L} W$
Maximum range	$\frac{V}{F} = \frac{\eta_{prop}}{c_p} \frac{1}{D}$ $D = minimum$	$\frac{V_0}{F} = \frac{V}{c_p D}$ $\left(\frac{D}{V}\right)_{min} \rightarrow \left(\frac{C_L}{C_D^2}\right) = max$
Maximum endurance	$F = \frac{c_p}{\eta_{prop}} DV,$ $(DV)_{min} \rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{max}$	$F = c_p D$ $D = minimum$

Steady climbing flight $\frac{dV}{dt} = 0, \frac{d\gamma}{dt} = 0$:

Equations

$$\sin\gamma = \frac{T - D}{W}, ROC = V\sin\gamma = \frac{TV - DV}{W} = \frac{P_a - P_r}{W}$$

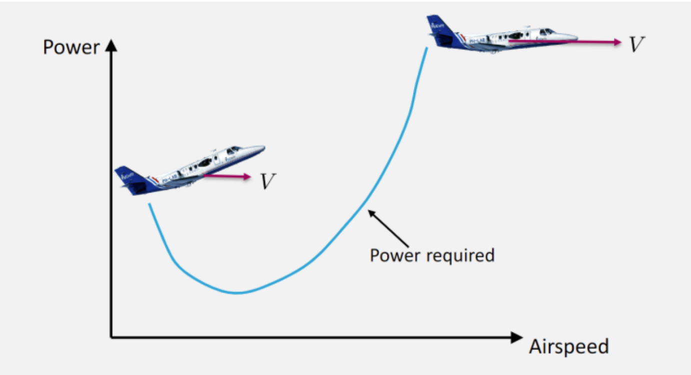
Route	**Propeller**	**Jet**
Maximum climb angle	-	$\sin\gamma_{max} = \frac{(T-D)_{max}}{W}$ $T = cons \rightarrow D_{min}$
Minimum descent angle (about how long the aircraft can stay, not how far) in gliding flight	$\sin\bar{\gamma} = \frac{C_D}{C_L}, \left(\frac{C_L}{C_D}\right)_{max}$	$\sin\bar{\gamma} = \frac{C_D}{C_L}, \left(\frac{C_L}{C_D}\right)_{max}$
Maximum ROC	$P_a = cons$ $ROC_{max} = \frac{(P_a - DV)_{max}}{W}$ $(DV)_{min} \rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{max}$	$ROC_{max} = \frac{(P_a - DV)_{max}}{W}$ $(DV)_{min} \rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{max}$
Minimum rate of descent in gliding flight	$ROC = -\frac{DV}{W},$ $(DV)_{min} \rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{max}$	$ROC = -\frac{DV}{W},$ $(DV)_{min} \rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{max}$

Important things:

- If one variable is constant, then this constant is usually given when analysing a problem.
- Minimum rate of descent: You minimize the rate of descent when you want to lose altitude slowly and stay airborne for as long as possible.
- Minimum glide angle: You minimize glide angle when you want to cover the longest horizontal distance with the least loss of altitude.
- Maximum climb angle: when you need to clear obstacles in a confined area or after takeoff.
- Maximum rate of climb: when you want to reach a higher altitude in the shortest possible time.

Lecture 13: Altitude effects

All performance diagrams and calculations were assumed the air density to be constant. But in reality, aircraft fly at various altitudes. Therefore, it is useful to know what the effects are on the performance diagram if the altitude increases. Recall a performance diagram:



Here, the power required is plotted as a function of airspeed. The airspeed depends only on the C_L if we assume that $\rho = constant$, which, in turn, depends on the angle of attack.

Assuming you want to fly higher with the same angle of attack, thus the same C_L, what would happen to the airspeed? Essentially, we pick a single point on the diagram and see where it ends up when the altitude increases.

Drag vs speed

Since we fly higher at a specific point on the performance diagram, C_L and C_D are constant and independent on altitude. Recall

$$D = \frac{C_D}{C_L} W$$

This is not dependent on the density, thus that means:

$$D_{h_1} = D_{h_2}$$

When we look at the airspeed equation:

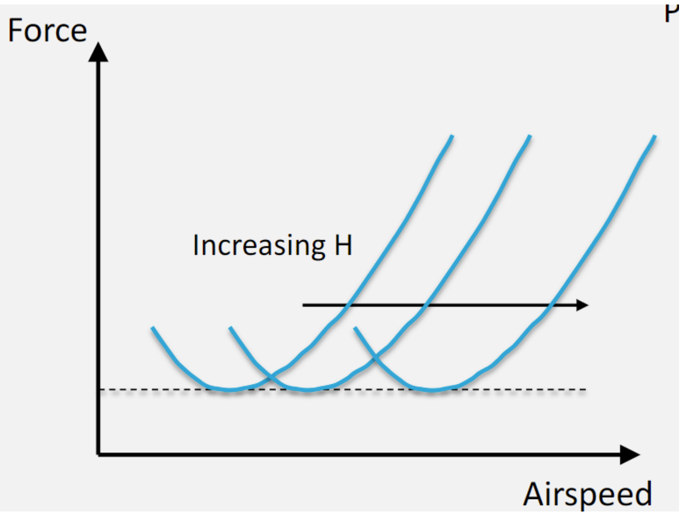
$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{max}}}}$$

Because all are constant except ρ, then

$$V_{h_1} = \sqrt{\frac{W}{S}\frac{2}{\rho_{h_1}}\frac{1}{C_{L_{max}}}} < V_{h_2} = \sqrt{\frac{W}{S}\frac{2}{\rho_{h_2}}\frac{1}{C_{L_{max}}}}$$

This relationship makes sense because, if we increase the altitude, ρ decreases, which means that airspeed needs to increase to maintain lift.

For drag and airspeed, this means that increasing altitude will shift the performance diagram to the right:



Power vs speed

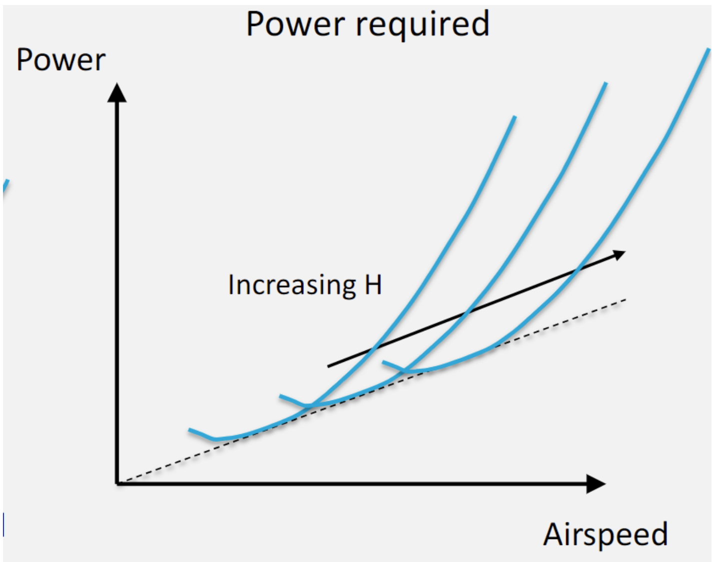
As for a power-airspeed diagram,

$$P_{h_1} = D_{h_1} V_{h_1}, \qquad P_{h_2} = D_{h_2} = V_{h_2}$$

If we know that drag stays constant then,

$$\frac{P_{h_1}}{V_{h_1}} = D_{h_1} = D_{h_2} = \frac{P_{h2}}{V_{h_2}}$$

This means that increasing the altitude, will not impact the ratio between the power and the airspeed, resulting in



Thrust

The altitude will impact the thrust given that temperature and air density decreases.

Recall

$$T = \dot{m}(V_j - V_0)$$

Decreasing the air density would decrease the mass flow

Colder air means that air density actually increases slightly, but the effect of density is still greater.

This means that aircraft can generate the most thrust at cold conditions at low altitude, because this would increase the air density by a lot.

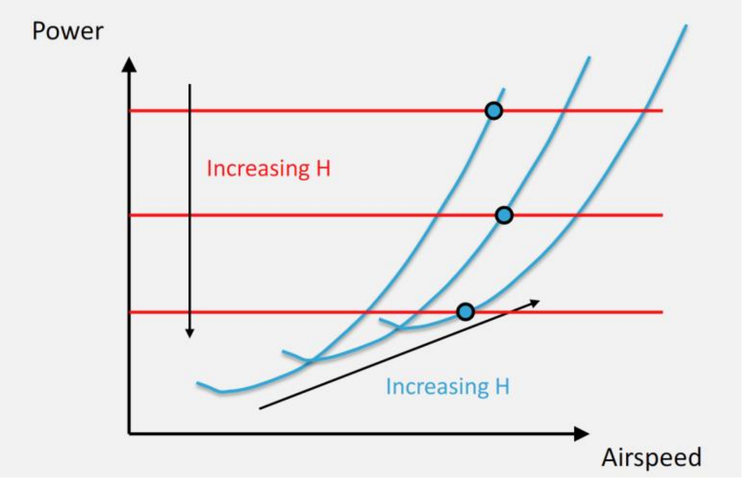
Generally the effect of altitude on the thrust is given by:

$$T = T_0 \left(\frac{\rho}{\rho_0}\right)^n$$

$$\frac{P_a}{P_{a0}} = \left(\frac{\rho}{\rho_0}\right)^n$$

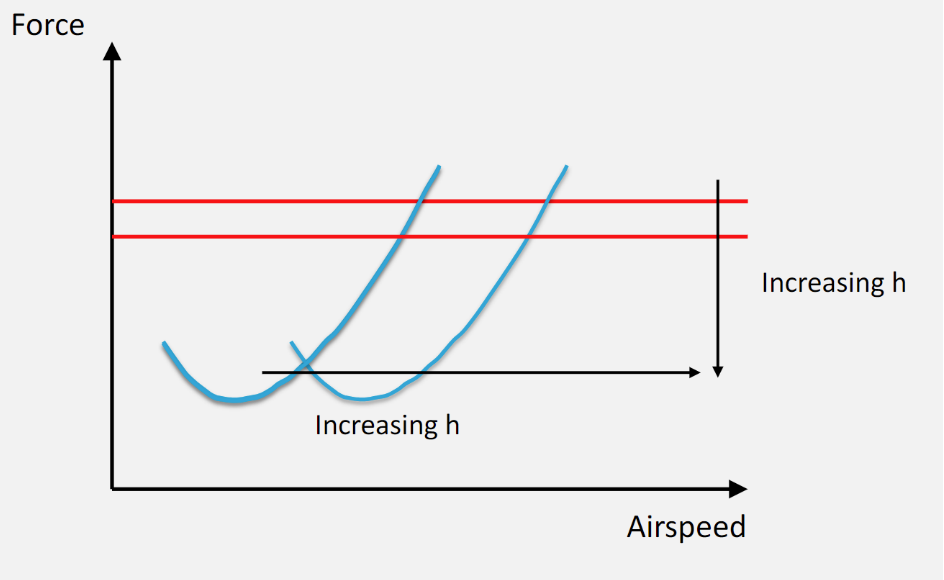
The factor n will vary for specific types of engine and will always be given. From the density, the height can be calculated.

Thus, in general, the thrust decreases with altitude.



Performance Limits

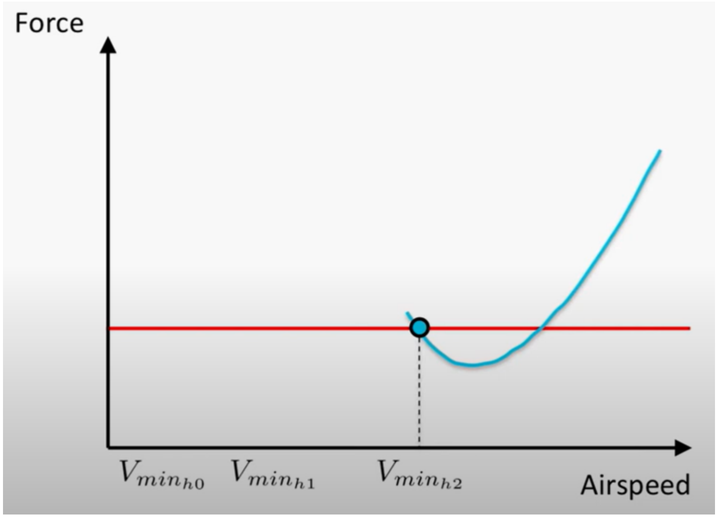
Summarizing the performance diagram for a jet aircraft:

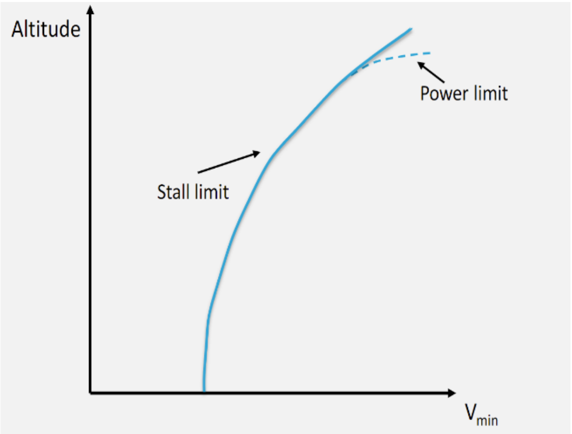


Minimum airspeed

If we increase the altitude, the thrust goes down and the drag shifts to the right, the minimum airspeed is when $C_{L_{max}}$. However, the thrust will shift downwards until the starting point for the drag becomes higher than the thrust. Then, the minimum airspeed is the first intersection between the thrust and the drag-curve.

This means that the minimum airspeed increases with increasing altitude.



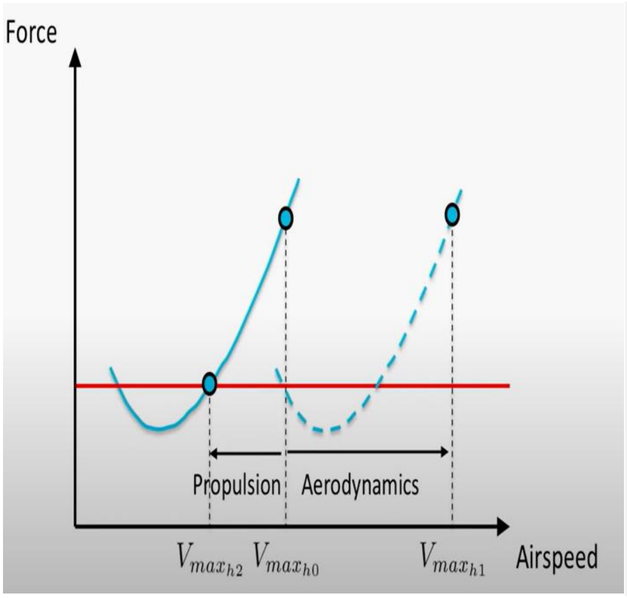


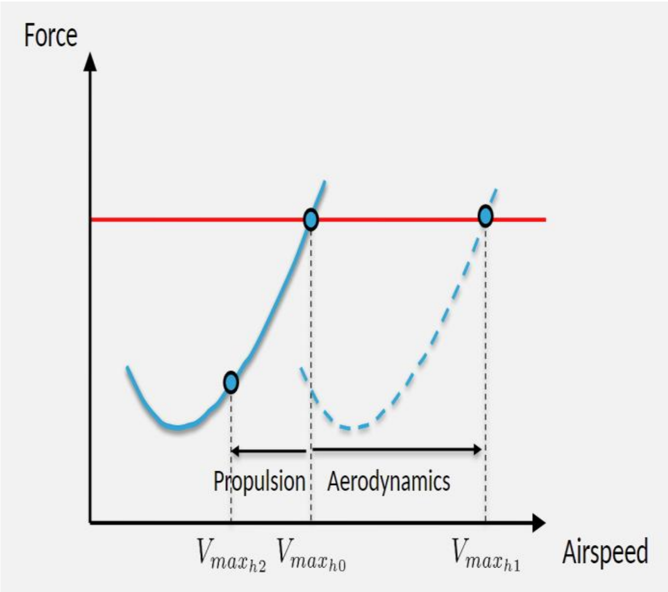
At some altitude, the curve has to be corrected according to the intersection between thrust and drag. So, then the thrust decreases even further.

Maximum airspeed

The shift to the right would mean that the maximum airspeed increases. However,

The decreasing maximum thrust according to $T = T_0 \left(\frac{\rho}{\rho_0}\right)^n$ will cause the thrust curve to move downwards. Therefore, the intersection between thrust and drag can either occur at a more to the left or more to the right. This means that the maximum airspeed due to altitude can either increase or decrease, depending on the type of engine according to the factor n.



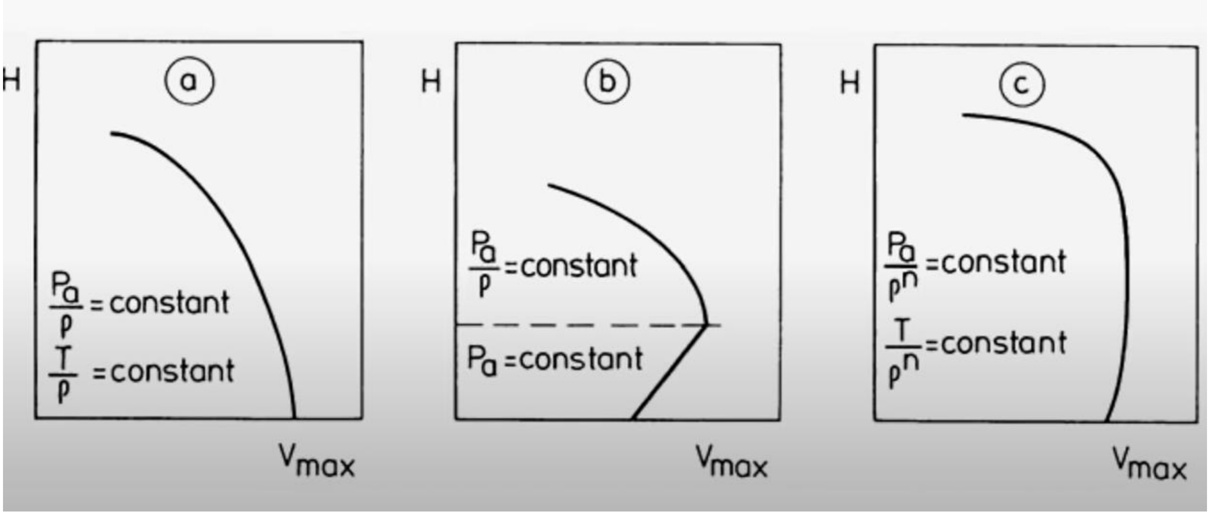


If the maximum airspeed increases with altitude, the dominant part that changes the performance diagram is the **aerodynamics.** Since the thrust curve will not decrease as much, the aerodynamics will determine the increase in maximum airspeed. The factor n is then very low.

Aerodynamics is dominant if the maximum airspeed increases.

The thrust settings are dominant if the maximum airspeed decreases.

Let's consider three different types of engines: piston engine, supercharged piston engine, turboprop. The factor n is unique for all three engines. This means that the maximum airspeed is different for all of them.



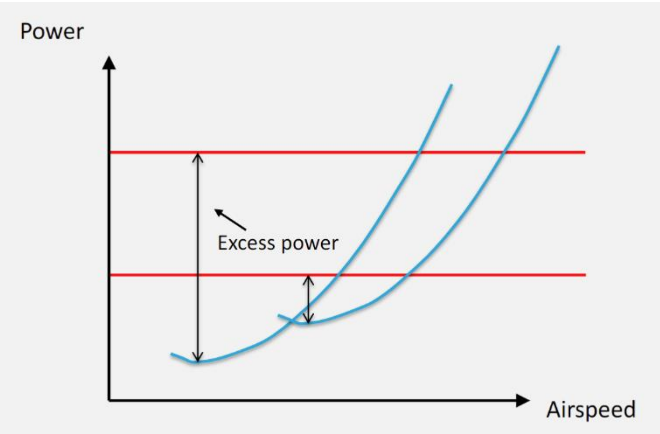
- The first case (piston engine) tells us that the maximum airspeed decreases as a function of altitude.
- The second case (supercharged piston engine) tells us that the maximum airspeed increases first and then decreases.
- In the final case (turboprop), the maximum airspeed stays constant for a while and then decreases.

Maximum rate of climb

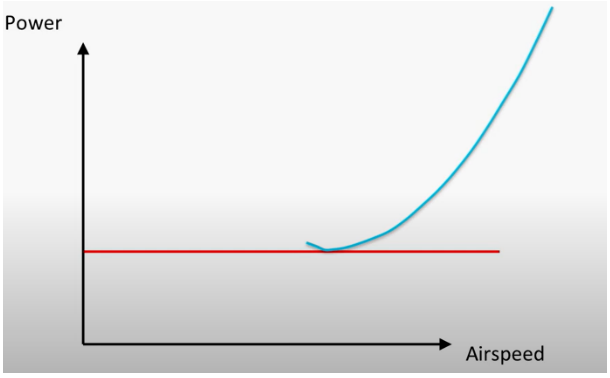
Recall the rate of climb

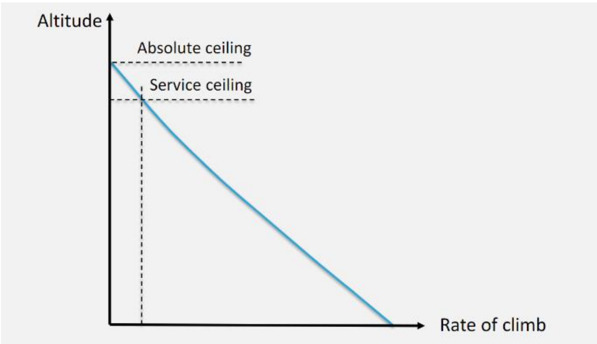
$$ROC = \frac{P_a - P_r}{W}$$

In the performance diagram, the maximum rate of climb is the maximum distance between $P_a - P_r$, or the $P_{r_{min}}$.



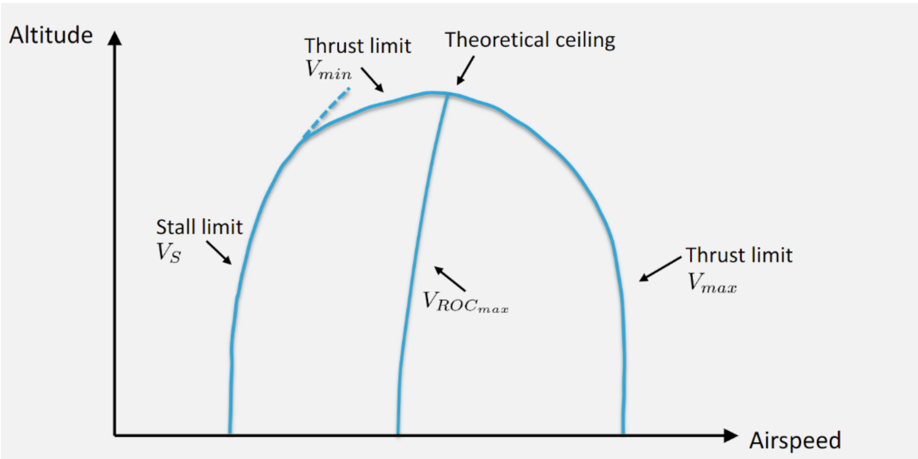
When we increase the altitude, $P_a - P_r$ decreases, thus the rate of climb decreases. The corresponding airspeed actually increases. The point at which the maximum rate of climb equals zero for a specific altitude is when $P_a - P_r = 0$.





The point at which the maximum rate of climb equals zero is called the absolute ceiling. This is the maximum altitude for which we can fly steadily. Going higher than that would mean that we descend.

The performance limits for minimum airspeed and maximum airspeed would look like this:



The theoretical ceiling is also the maximum altitude. This means that at this altitude, there is only one airspeed is possible. Aircraft that want to fly very high need to have:

- High aspect ratio.
- Low C_{D_0}
- Propulsion systems designed for high altitude.

To determine this maximum altitude based on some aircraft data, we must determine what happens to the thrust. Assuming that the thrust decreases parallel with the density. Since this altitude is at the stratosphere, the density starts at the stratosphere level:

$$\frac{T}{T_0} = \frac{\rho}{\rho_0}$$

Knowing that $T = D$,

$$\frac{C_D}{C_L} W = T \quad \rightarrow \quad \frac{1}{T_0} \frac{C_D}{C_L} W = \frac{\rho}{\rho_0}$$

The thrust at stratosphere level as well as the density are all constant. That means that flying high means that the air density must be as low as possible, ρ_{min}

$$\rho_{min} = \frac{\rho_0}{T_0} \left(\frac{C_D}{C_L}\right)_{min} W$$

This gives

$$C_L = \sqrt{\frac{C_{D\,0}}{k_2}}$$

This means that

$$C_D = C_{D_0} + \frac{\frac{C_{D\,0}}{k}}{k} = 2C_{D_0}$$

Which gives

$$\rho_{min} = \frac{\rho_0}{T_0} \frac{2C_{D_0}}{\sqrt{C_{D_0} \pi A e}} W = \frac{\rho_0}{T_0} \sqrt{\frac{C_{D_0}}{A e \pi}} W$$

This gives us insight into the design of a high-altitude aircraft:

- High aspect ratio
- Low C_{D_0}
- Propulsion system that is able to fly at low density

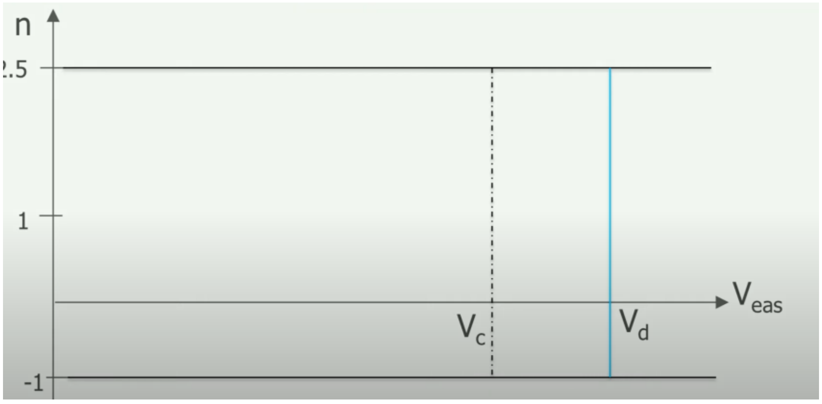
Operational Limits

This graph shows what the aircraft is able to do based on the propulsion system, aerodynamic performance and aircraft weight. This does not mean that the aircraft is allowed to fly at these conditions. There are some operational limits that need to be taken into account.

Manoeuvre loads:

During any flight, the pilot will intentionally perform manoeuvres. In this case, the lift will be either larger or smaller than the weight. The load factor is the ratio between lift and weight and expressed in $g's$. The airframe will have additional load due to this manoeuvre which needs to be counteracted. The flight-manoeuvering is a graph with the load-factor on the y-axis and the Equivalent airspeed on the x-axis. The type of manoeuvers vary. Consider a simple jet airplane in which limit positive load factor of 2.5 and a limit negative load factor -1. Within that region are the usual types of manoeuvers for jet airplanes.

The cruise speed is speed the aircraft flies steadily with the best conditions. The design dive speed is the speed at limiting speed at which aircraft is free of vibrations and buffeting. The aircraft must not go faster than the design dive speed. The design cruise speed is 25% greater than the cruise speed.



The design dive speed typically will be encountered in a dive. It is the maximum speed in which the aircraft can be safely flown at a dive. If one conducts a manoeuvre with a negative g, it means the plane will go in a steeper dive. That means the area region between V_c and V_d and the negative g cannot be encountered.

For lower airspeed, stall is important to consider. The stall speed is, essentially the minimum airspeed at $C_{L_{max}}$.

$$L = nW \rightarrow \frac{1}{2}\rho V^2 S C_L = nW$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}\sqrt{n}$$

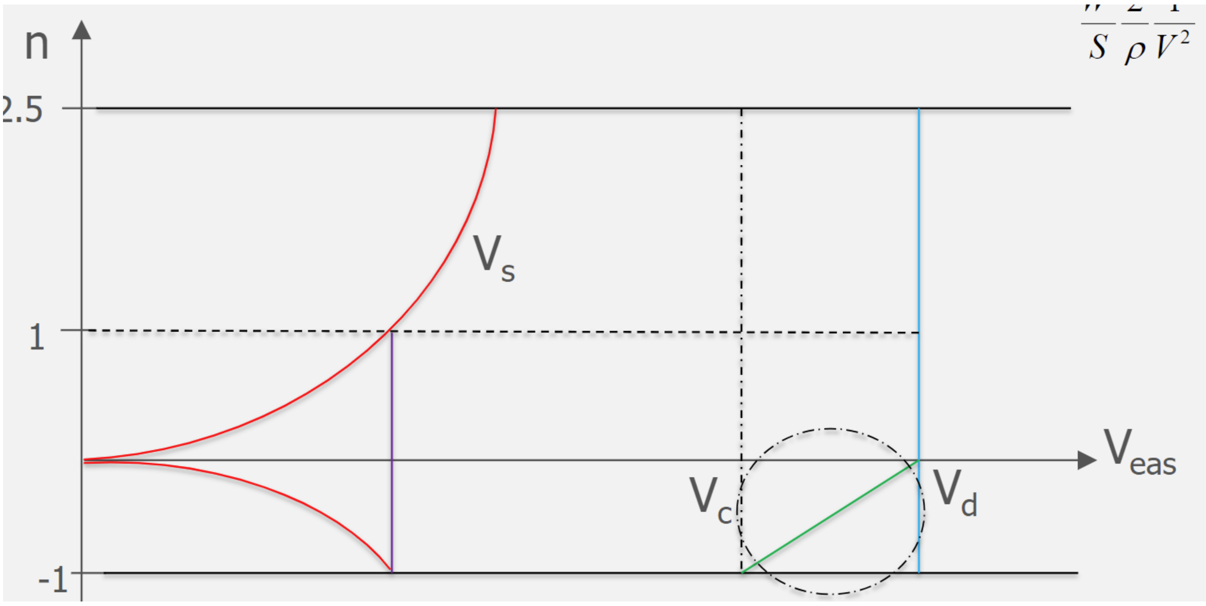
The minimum airspeed at cruise speed, $n = 1$ can easily be calculated, we call this $V_{min,cruise}$

Thus,

$$V_{min}(n) = V_{min,cruise}\sqrt{n}$$

For load factors that are larger than 1, so when the aircraft is maneuvering, the stall speed or minimum airspeed is a bit higher. So that means that the aircraft must fly faster.

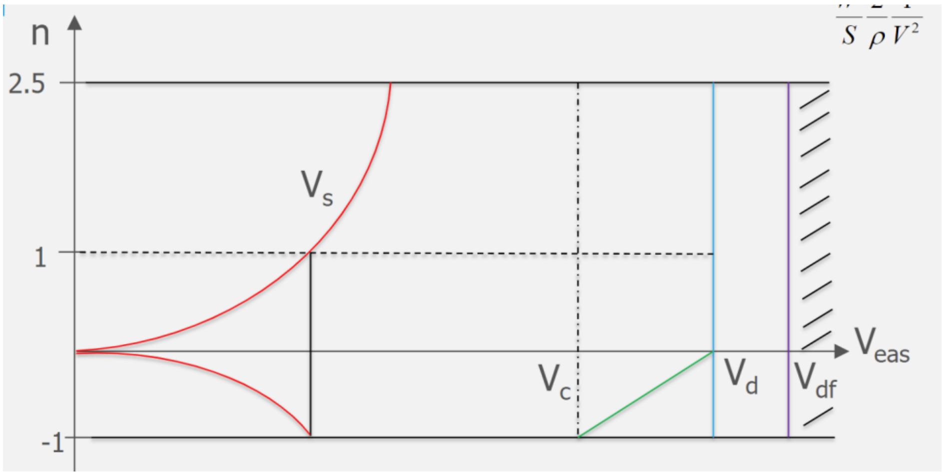
We can see this in the diagram:



The region between the line are the airspeeds in which the aircraft stays safe during all possible maneuvers.

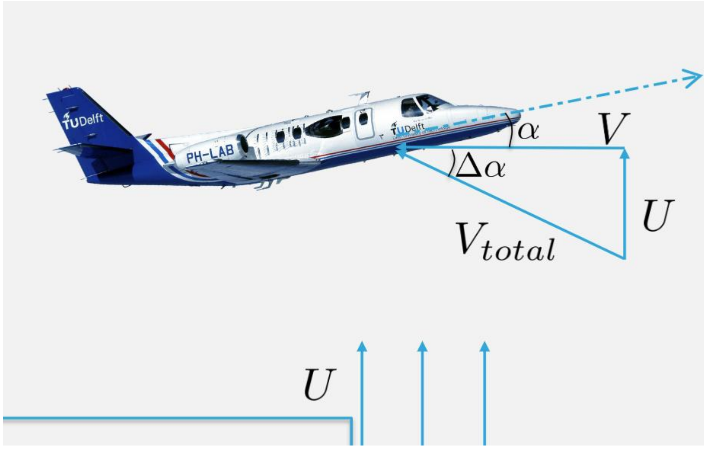
Aeroelastic effects

If an aerodynamic load acts on wing, it will deform, which means that the wing will have a different orientation with respect to the airflow. This different orientation changes the airload. If we increase the airspeed, the aerodynamic load increases. There is a speed in which aerodynamic load causes the wing to oscillate, which is very dangerous, also known as the flutter speed. The flutter speed is usually 15% larger than the design dive speed. To include this:



Gust loads

A gust is a sudden short, strong rush of wind. The structural design of an aircraft must cope gusts and turbulence that are equal to 25 ft/s. A gust will change the angle of attack.



The load factor is given by

$$n = \frac{L}{W}$$

With n expressed as the amount of $g's$

For cruise flight, $n = 1\ g$. A gust would change the lift, because the angle of attack increases, which means that the change in load factor

$$\Delta n = \frac{\Delta L_{gust}}{W}$$

$$\Delta L_{gust} = \Delta C_{L_{gust}} \frac{1}{2} \rho S (V^2 + U^2)$$

$$\Delta \alpha = \tan\left(\frac{U}{V}\right) \rightarrow \Delta C_L = \frac{dC_L}{d\alpha} \tan\left(\frac{U}{V}\right)$$

This means that

$$\Delta L_{gust} = \frac{dC_L}{d\alpha} \tan\left(\frac{U}{V}\right) \frac{1}{2} \rho S (V^2 + U^2) \approx \frac{dC_L}{d_\alpha} \frac{U}{V} \frac{1}{2} \rho V^2 \approx \frac{dC_L}{d_\alpha} U \frac{1}{2} \rho V$$

$$\Delta n = \frac{\frac{dC_L}{d_\alpha} \frac{U}{V} \frac{1}{2} \rho V^2}{W}$$

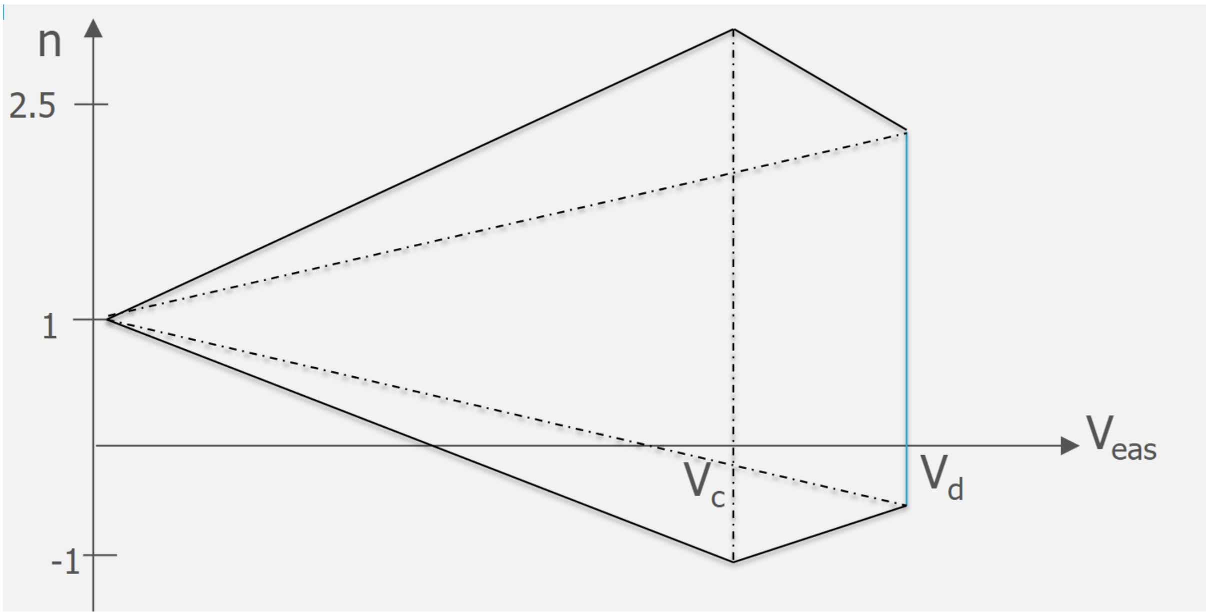
The airframe is structurally designed to withstand a certain load factor, so this load factor is a maximum load factor. The maximum load factor here only depends on the airspeed, with the others being constant. The airspeed must be below this expression, this is one operational limit:

$$V < \Delta n_{max} UV \frac{2}{\rho} \frac{W}{S} \frac{1}{U \frac{dC_L}{d\alpha}}$$

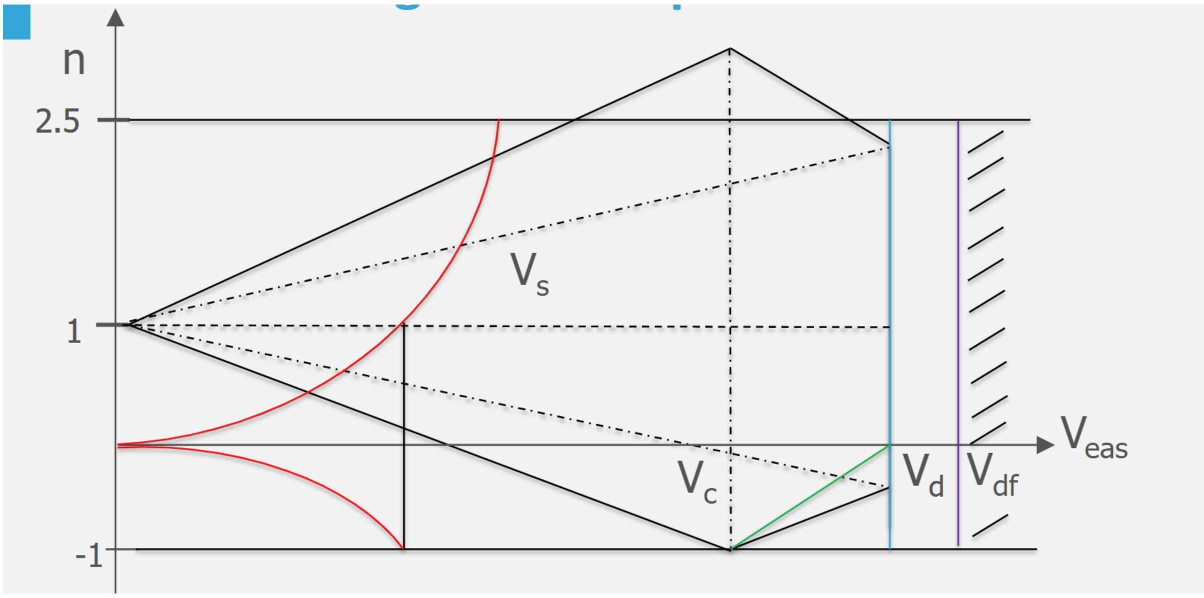
This airspeed changes when we fly higher.

The gust envelope can be drawn. The airworthiness specifications have specified the gust to be considered below or equal to the design cruise speed for high airspeeds. For a constant U, Δn. For a constant U, the Δn as a result of a gust relative to the condition where the load factor equals 1 becomes a straight line. At V_c the gust load is maximum.

When the gust speed is lower, the gust needs to be considered for the V_d. This is an additional graph.

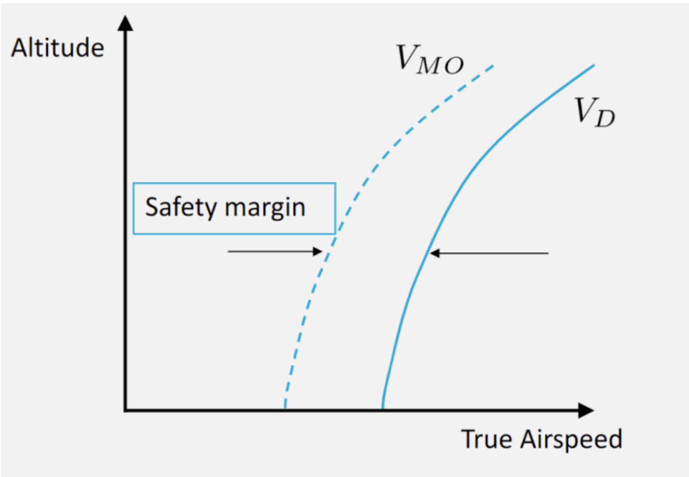


Combining both figures



These combined limits are those that should be considered when the aircraft structure is designed.

The flight envelope indicates the equivalent airspeed so that it applies for every altitude. The design dive speed changes with altitude. So, flying higher means that the design dive speed increases, which is why the flight envelope becomes slightly larger.



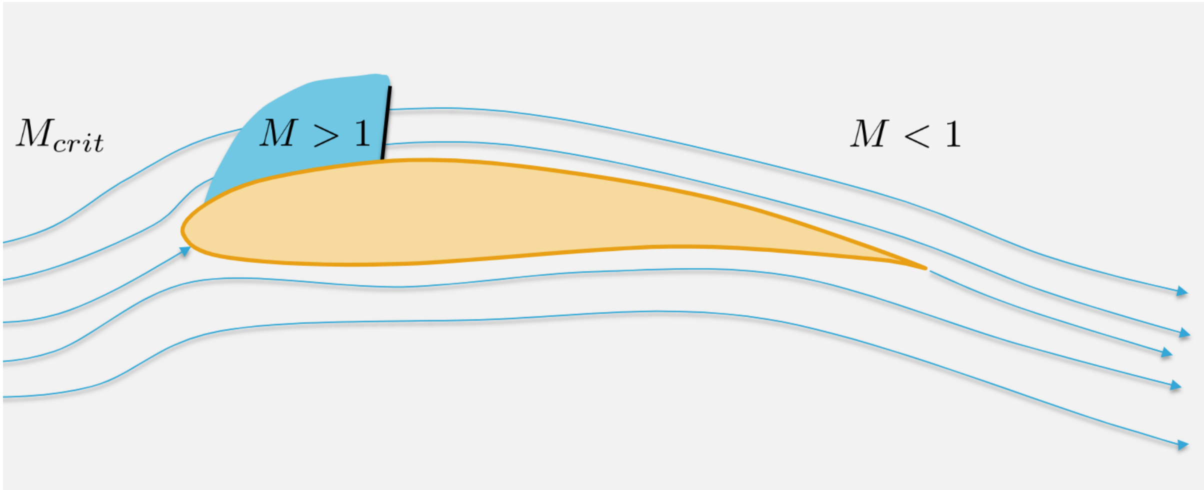
There is also a safety margin for assurance.

Maximum Mach number

$$C_D = C_{D_0} + k_1 C_L + k_2 C_L^2$$

If the mach number increases, the constants gradually increase.

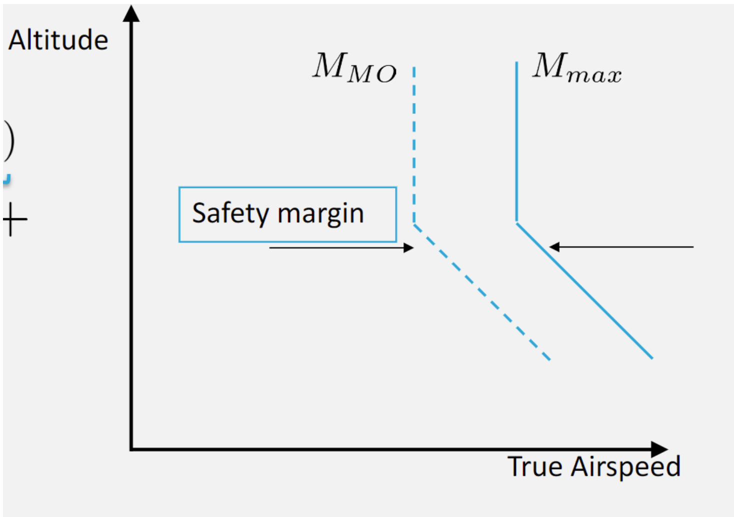
Even if the airspeed does not fly at supersonic speeds, it can still happen locally at the airfoil. We know that from Bernoulli's equation, the speed of airflow around the camber of the airfoil is higher than the airspeed. It could be the case that in this region, the airflow creates a supersonic region.



At mach numbers close to 1, shock waves start to appear which creates oscillations of the airfoil, that could be dangerous. Furthermore, the aerodynamic centre heavily shifts which causes a severe pitch moment. Thus, for civil jet aircraft, it is dangerous to fly at high mach numbers. That is why airplanes have maximum mach numbers for safe flight.

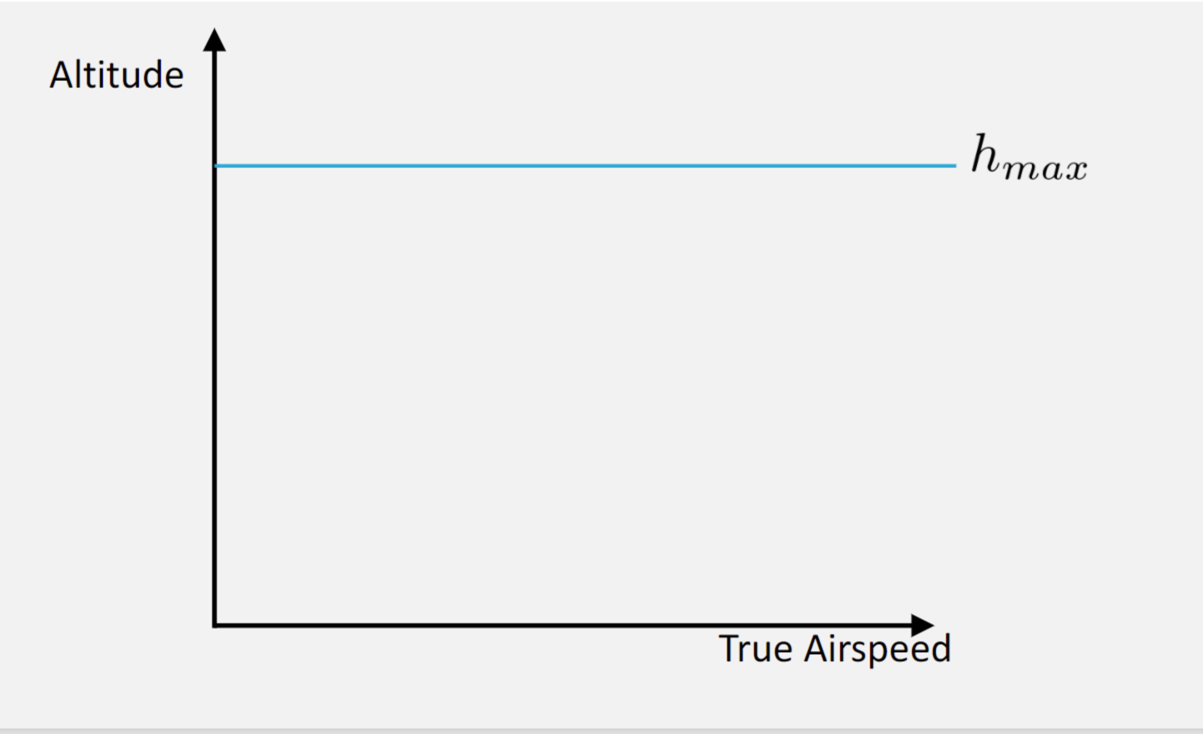
When flying higher, the temperature decreases.

Since mach number is a function of temperature, the corresponding airspeed must decrease with altitude for constant mach number until the maximum mach number is achieved. Again, a safety margin has been defined.



Pressurized cabin

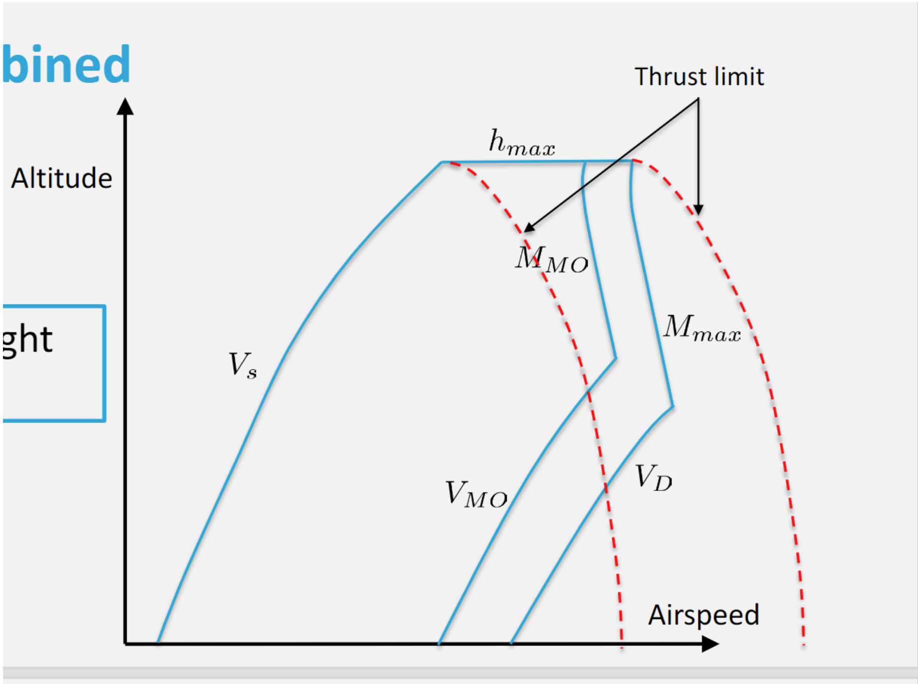
The cabin is pressurized for the safety of the passenger and loads, which means that the pressure at the cabin is higher than the pressure of the air. The difference between these pressures create stresses in the fuselage structure. There is a maximum pressure differential, so there is a maximum altitude in which the aircraft can fly before the pressure differential becomes too large. This is the maximum flight altitude.



Combining all performance limits:

The performance limits can depend on the weight of the aircraft as well.

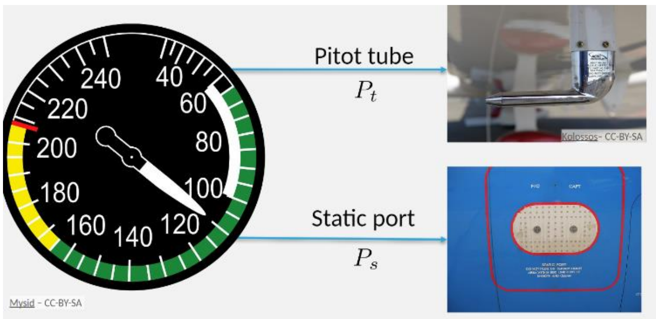
This picture gives us the altitude and airspeed to which aircraft is constrained for a specific aircraft weight.



Mach number = constant because of eq. airspeed, thus, airspeed decreases with maximum mach number.

All these boundaries vary with altitude, which is quite complex. For a pilot to monitor these limits,

- The stall speed increases with altitude and is dependent on the aircraft weight according to $V_{min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{max}}}}$. To monitor this stall speed, the pilot has an airspeed indicator, which measures the equivalent airspeed using a pitot tube. The total pressure and static pressure is measured. Assuming sea level density, the equivalent airspeed equals

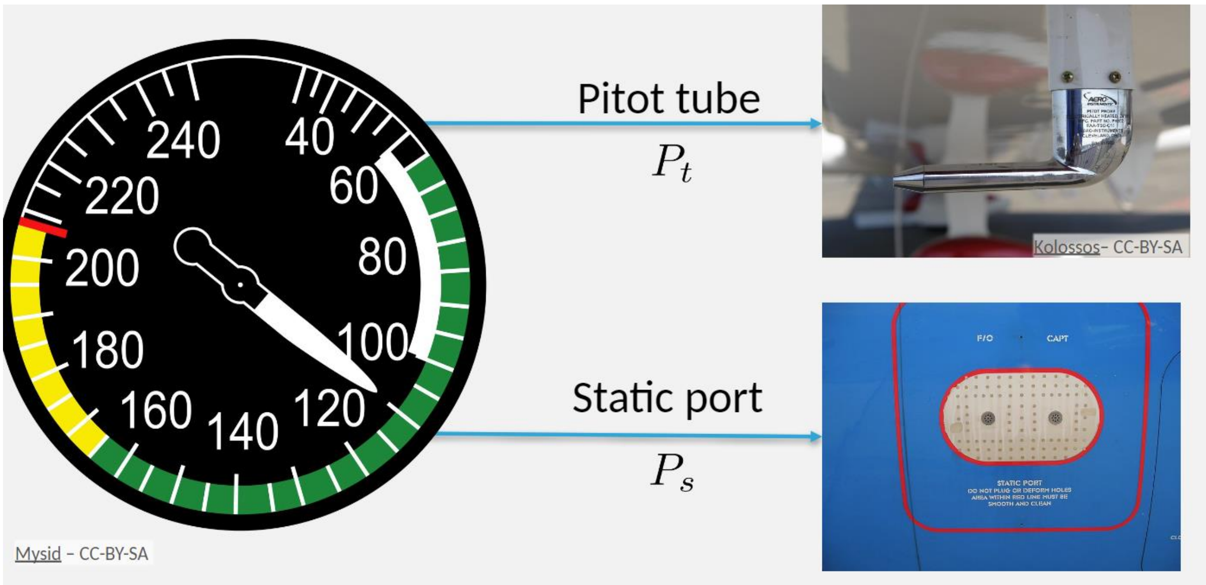


$$V_{EQ} = \sqrt{\frac{2(p_t - p_s)}{\rho_0}}$$

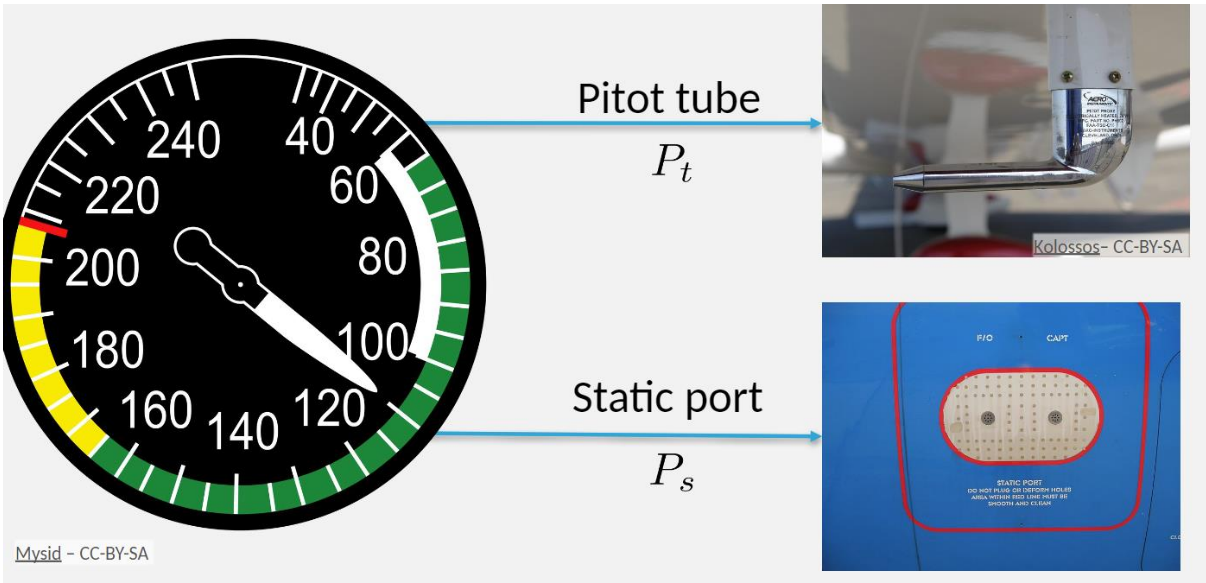
So, the minimum equivalent airspeed is independent of altitude:

$$V_{E_{min}} = \sqrt{\frac{W}{S}\frac{2}{\rho_0}\frac{1}{C_{L_{max}}}}$$

The true airspeed is always higher than the equivalent airspeed, which means that the stall speed limit is true for both airspeed and equivalent airspeed. This speed is indicated in the green boundary.

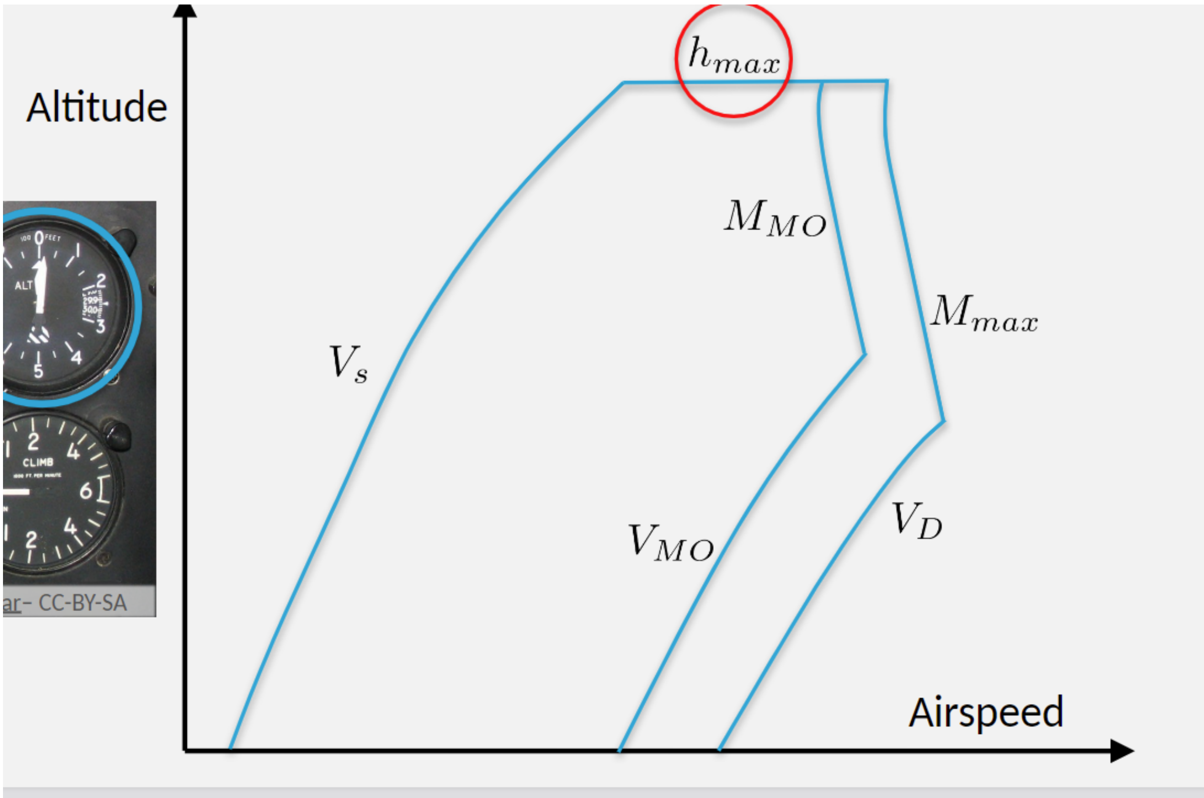


- The airspeed indicator measures the dynamic pressure, which is what the wing feels.



The maximum operating airspeed is a limit which is also constant when expressed as equivalent airspeed, this is yellow boundary.
Gust operating limit indicates that lowering the airspeed will prevent the impact of the gust. The yellow region is the region with risks at bad weather, such as gust loads.

- For the maximum altitude, there is a altimeter which measures the static pressure, which is an indication for altitude. It always assumes sea-level conditions, which is not always true. Thus, the altimeter is an indication for Flight-level. This will not impact the true maximum altitude.



- For the maximum operating mach number. The maximum operating mach number needs to be considered. The mach meter measures this the mach number.

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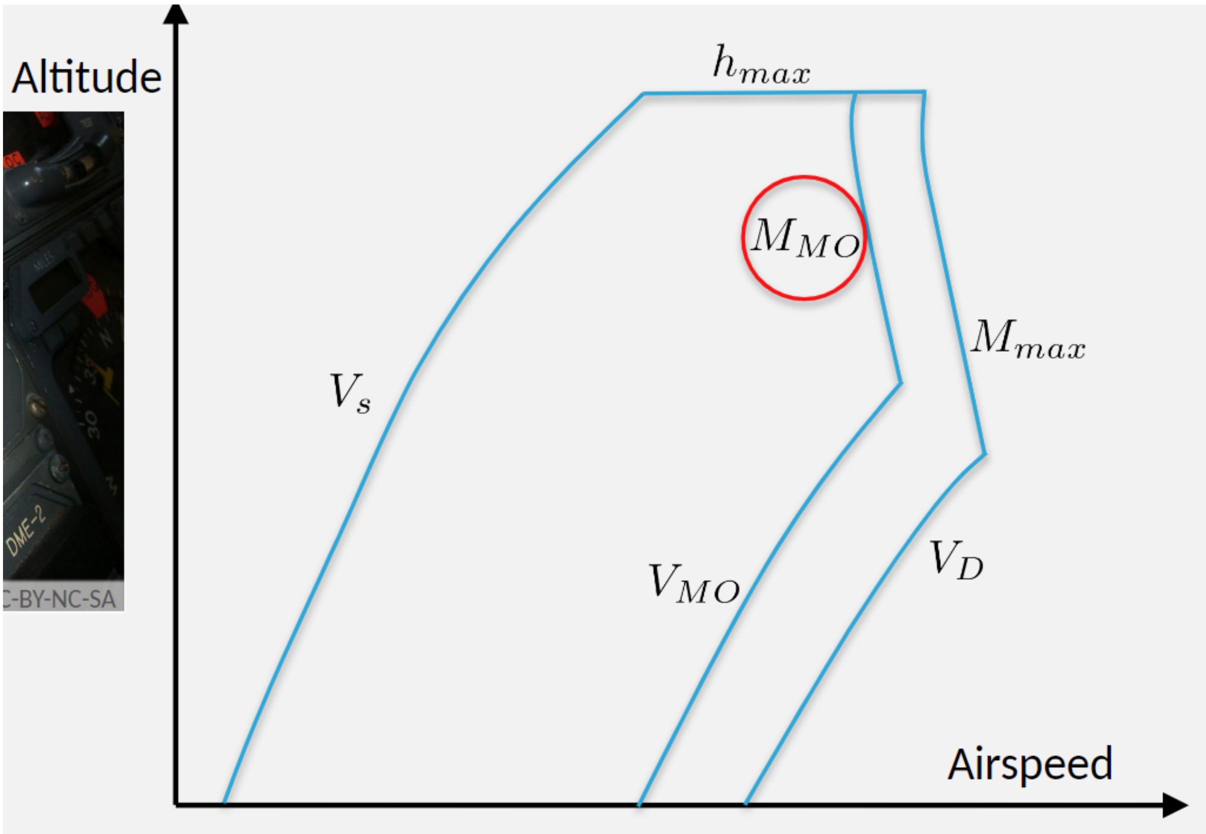


Tips:

- Steady, meaning dV/dt =0. Proving if it's steady, determine whether dV/dt =0.
- Power limiting behavior, $T = D_{min} \rightarrow D_{min} = \frac{C_D}{C_{L_{max}}} W$
- Total $n_t = n + \Delta n$
- $\frac{dC_L}{d\alpha}$ is always in radians.
- If one variable is constant, then this constant is usually given when analysing a problem.
- Minimum rate of descent: You minimize the rate of descent when you want to lose altitude slowly and stay airborne for as long as possible.
- Minimum glide angle: You minimize glide angle when you want to cover the longest horizontal distance with the least loss of altitude.
- Maximum climb angle: when you need to clear obstacles in a confined area or after takeoff.
- Maximum rate of climb: when you want to reach a higher altitude in the shortest possible time.
- Increasing airspeed → aerodynamics of the aircraft
- Decreasing airspeed → engine settings of the engine

Lecture 1 Aerodynamics

Aerodynamics is the science that deals with the flow of air.

Pressure

When you hold your hand outside the window of a moving automobile with you palm, you can feel the air pressure exerting a force and tending to push your hand. **Pressure** is the force per unit area. Air molecules are striking the surface of your hand and transferring some of their momentum to the surface.

Note that pressure can still be defined if there is not surface. In fact, it is defined as point in the gas or a point on a surface →

$$p = \lim_{dA \to 0} \left(\frac{dF}{dA} \right)$$

The pressure is the limiting form of the force per unit area where the area of interest approaches to zero.

Density

The density of a substance is the mass of that substance per unit volume. Again, you do not need a volume. The volume of interest approaches 0, ρ is the mass inside this volume:

$$\rho = \lim_{dV \to 0} \left(\frac{dm}{dV} \right)$$

Temperature

Temperature is a measure of the average kinetic energy of the particles in the gas:

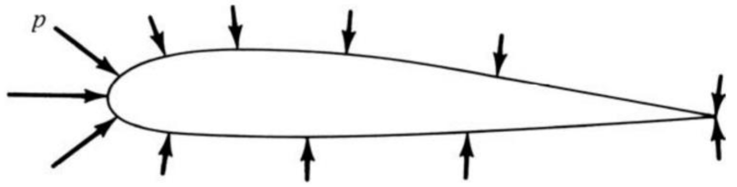
$$E_k = \frac{3}{2} k_B T$$

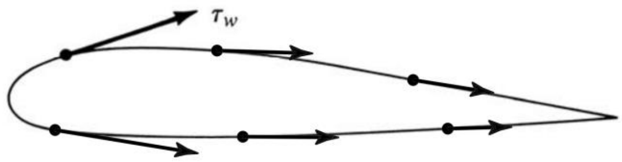
Velocity

The velocity at any fixed point in a flowing gas is the velocity of an infinitesimally small fluid element as it sweeps through that point.

Probably the most practical consequence of flow of air over an object is that an object experiences an aerodynamic force. Flow around an airfoil stems from:

1. Pressure distribution on the surface
2. Shear stress (friction) on the surface.





Contrary to pressure, the shear stress is due to the frictional effect of flow ‘rubbing’ against the surface as it moves around the body. It is the force per unit area tangentially on the surface.

A perfect gas is a type of gas in which intermolecular forces are negligible. The relation between density, pressure and temperature is:

$$p = \rho R T$$

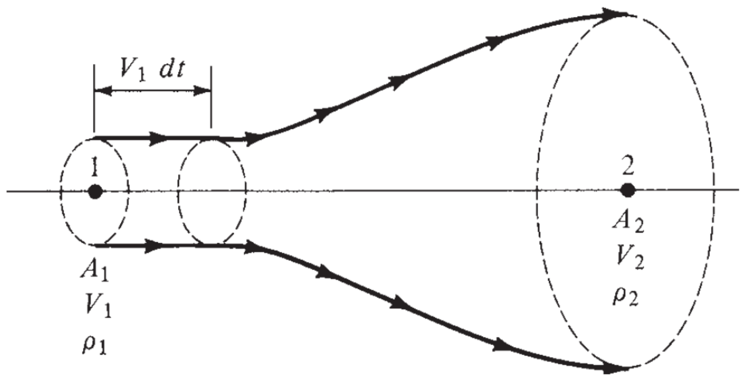
With $R = \frac{R_0}{M_{gas}}$. This means that the constant R is unique for every type of gas.

The specific volume v is defined as volume per unit mass:

$$v = \frac{1}{\rho} = \frac{V}{m}$$

In a flowing gas, mass can neither be created nor destroyed.

Consider all the streamlines that go through the circumference of the circle, these form a streamtube. We can see that the cross-sectional area may change, in moving from point 1 to point 2. The mass flowing through the streamtube is confined by the streamlines of the boundary, much like a wall.



Here, the flow of mass is constant across the whole tube. This means that

$$\dot{m} = constant = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

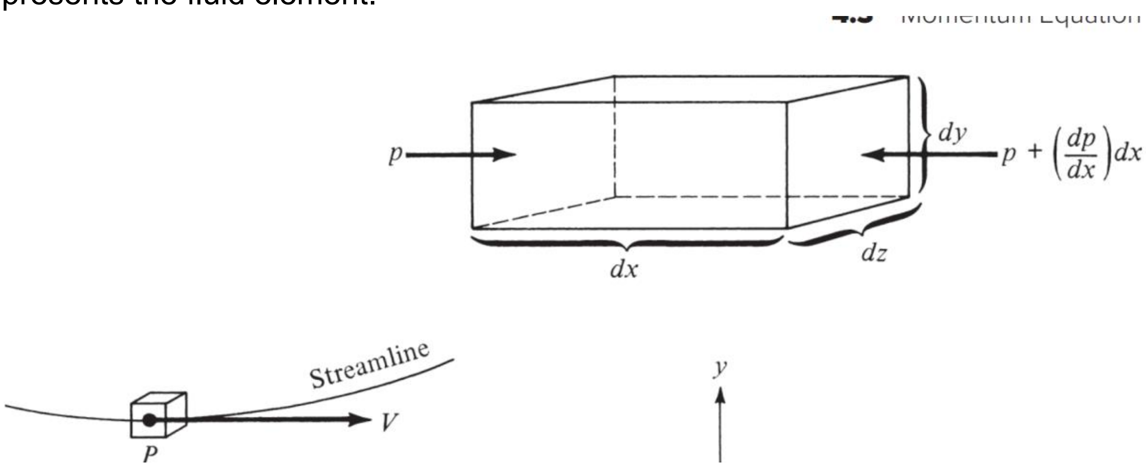
Compressible flow is similar to squeezing the element hard enough with some pressure, the volume of the element of matter will decrease. Its mass will stay the same, which changes the density of the material.

Incompressible flow is a flow in which the density of the fluid element stays constant. This is just a myth, can be a good simplified model, which means that

$$A_1 V_1 = A_2 V_2$$

Lecture 2 Bernoulli and Euler

To derive Bernoulli's equation, we can use a model of an infinitesimally box which represents the fluid element.



There are three forces acting on the fluid element:
- Pressure acting in the x-direction on all six faces.
- Frictional shear acting tangentially on all six faces of the element
- Gravity acting on the mass inside the element.

However, we will ignore the presence of frictional forces and gravity.

The pressure on the left face is p. The pressure varies from point to point in the flow due to the changing velocity. Hence, the change in pressure per unit length , multiplied by a distance $dx \rightarrow p + \frac{dp}{dx}dx$. Therefore the net force in the x-direction is

$$F_x = pA - (p + dp)A = pdy\Delta z - pdydz - dpdydz = -dpdydz$$

Applying Newton's second law

$$F_x = ma = \frac{\rho V dv}{dt} = \rho dxdydz \frac{dv}{dt}$$

Equating both equations:

$$\rho dxdydz \frac{dv}{dt} = -\rho dydz$$

$$\rho \frac{dx}{dt} dV = -dp$$

$$dp = -\rho V dV$$

This equation is a differential equation, hence it describes the phenomena in an infinitesimally small neighbourhood around the given point P. To consider two points, far removed from each other, in an incompressible environment. To relate p_1 and V_1 to p_2 and V_2, integration is required.

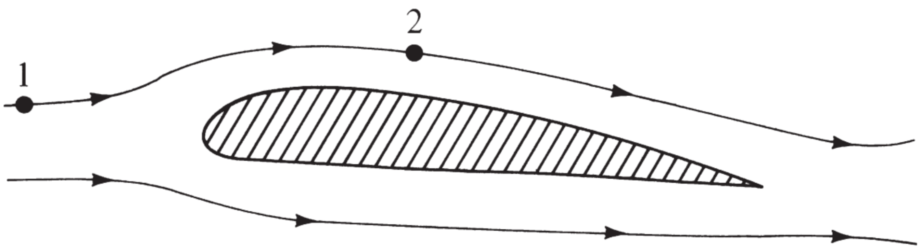


Figure 4.7 Two points at different locations along a streamline.

$$\int_{p_1}^{p_2} dp + \rho \int_{V_1}^{V_2} V dV = 0$$

$$p_2 - p_1 + \rho \left(\frac{1}{2} V_2^2 - V_1^2\right) = 0$$

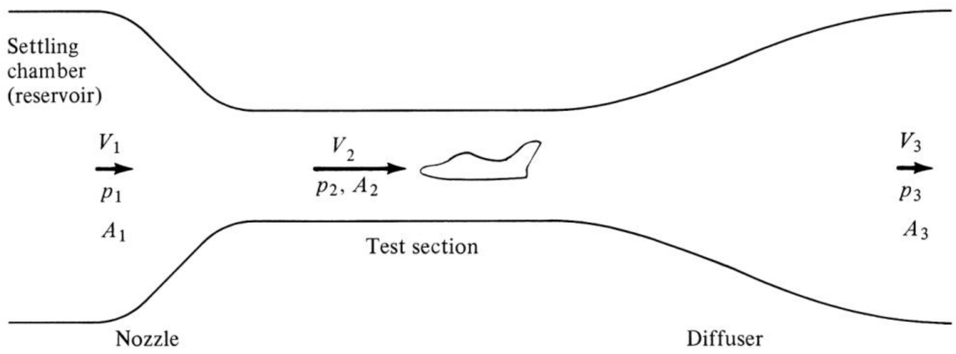
$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 = constant$$

This simple algebraic equation only works if the flow is:

- Inviscid (frictionless)
- It is a point property, meaning that it relates properties between different points along a streamline.
- Incompressible.

Low-speed subsonic wind tunnels

Wind tunnels are ground-based facilities designed to produce flows of air that simulate natural flows occurring outside the laboratory. The essence is drawn:



The airflow with pressure p_1 enters the nozzle with V_1. $A_2 < A_1$ at the test section. If we assume the flow to be incompressible:

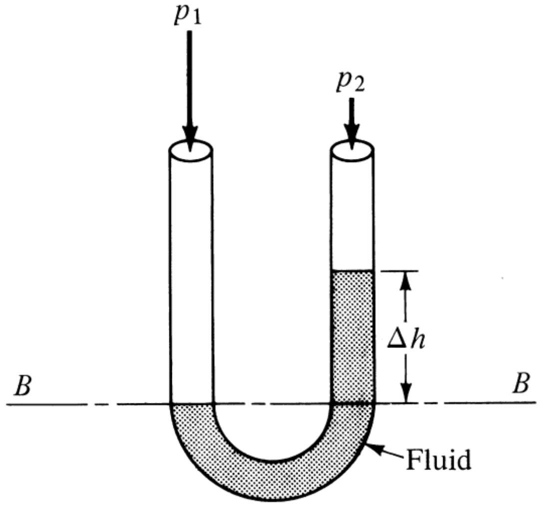
$$V_2 = \frac{A_1}{A_2} V_1$$

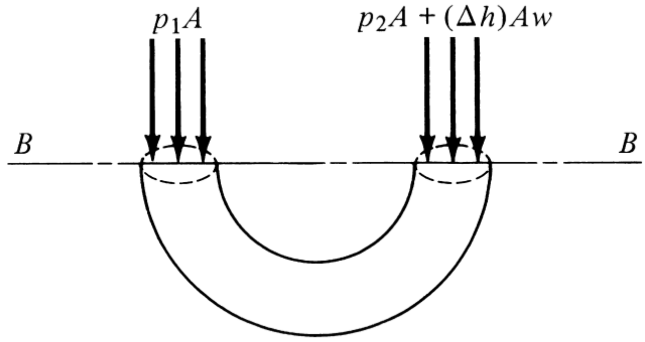
The corresponding pressures are expressed from Bernoulli's equation:

$$\frac{1}{2}\rho V_1^2 + p_1 = \frac{1}{2}\rho V_2^2 + p_2 \rightarrow V_2 = \sqrt{\frac{2}{\rho}(p_1 - p_2) + V_1^2}$$

$$V_2 = \sqrt{\frac{2(p_1 - p_0)}{\rho(1 - \left(\frac{A_2}{A_1}\right)^2}}$$

To calculate V_2, we need therefore the pressure difference $(p_1 - p_2)$. A manometer is used for this.





A simple manometer is a tube in an ‘U’ shape, filled with a fluid. In this situation one side of the U tube is connected to a certain pressure p_1. The other side is connected to a certain pressure p_2. Because p_1 is larger than p_2, the fluid at the right side goes up a height Δh. The force on the left exerted on the fluid is $p_1 A$. The force on the right, at the same height (section BB), is caused by the pressure p_2 and the weight difference of the fluid due to the height, $F_{right} = p_2 A + \rho g A \Delta h$. Fluid is stationary, therefore

$$p_1 A = p_2 A + \rho g A \Delta h \rightarrow p_1 - p_2 = \rho g \Delta h$$

Hence, the pressure difference is only governed by the displacement Δh. If the left side of the tube is connected to the section of p_1, and the other side of the tube is connected to the high speed section with p_2, then the difference can be measured. If the liquid is water then,

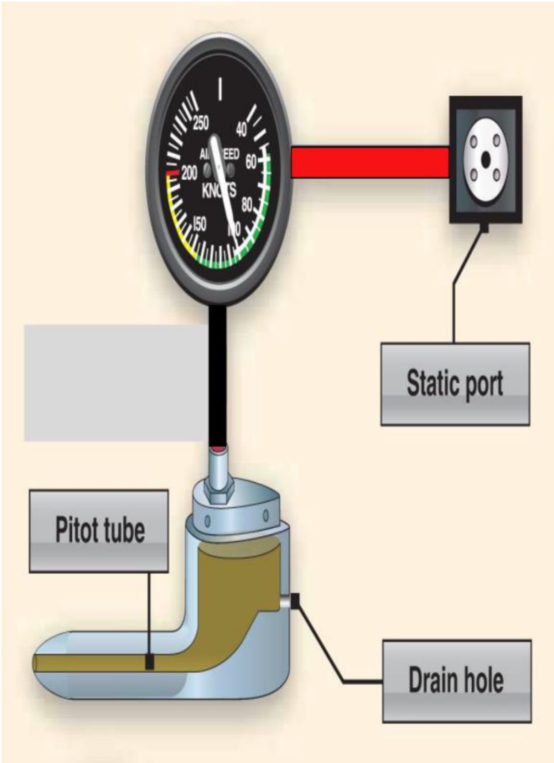
$$\Delta p = 9.81 \cdot \Delta h$$

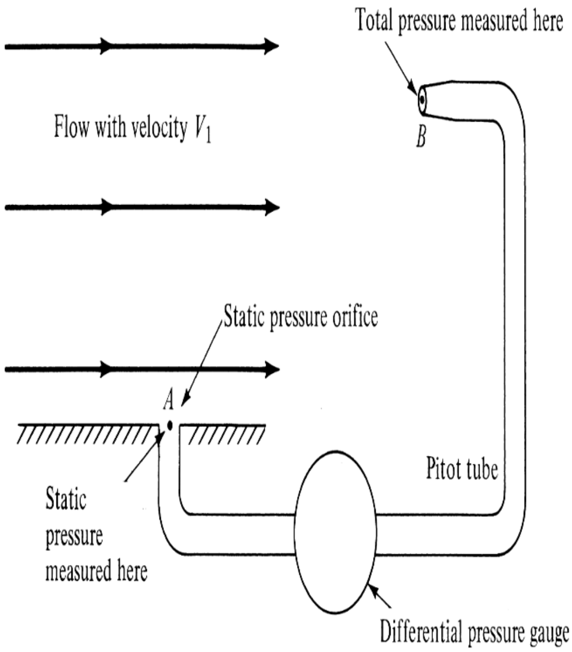
Manometers can be used to determine the pressure distribution of an airfoil. If the manometers are connected with points on the upper surface area of the aerofoil and the atmosphere, the pressure can be determined.

Measurement of airspeed

The static pressure, is a result of intermolecular reactions within the moving air (relative speed = 0) (perpendicular to the flow), and the total pressure is when an external object experiences both the speed and the intermolecular reactions. Supposedly, if we were moving along with the flow at a point, the pressure at that point is the static pressure.

The difference is the dynamic pressure. The total pressure is the pressure that would exist if the flow were slowed down to zero velocity. As a fluid element is brought to rest, pressure, the pressure will significantly increase according to the conservation laws.





A pitot tube is connected with the static port. The static port is placed perpendicular to the airflow, meaning it measures the static pressure, which is just p_{atm}. The opening at point B, is closed. At point A, where the static pressure is measured, the opening is open and it measures the random motion of the airflow. At point B, where the opening is closed, the airflow has no choice but to stop at the designated point B. At V=0, the p_{tot} is measured. Using Bernoulli's equation,

$$p_s + \frac{1}{2}\rho V_1^2 = p_{tot}$$

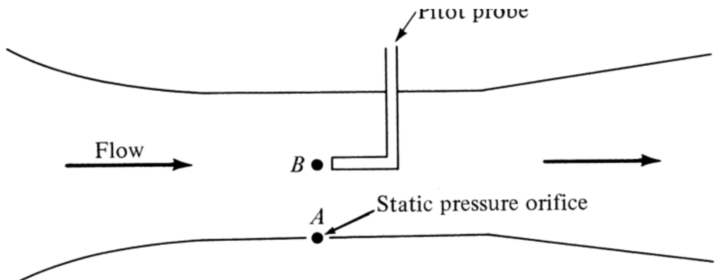
Where $q = \frac{1}{2}\rho V_1^2$, the dynamic pressure for flows of all types. In essence,

$$p_{tot} = p + q$$

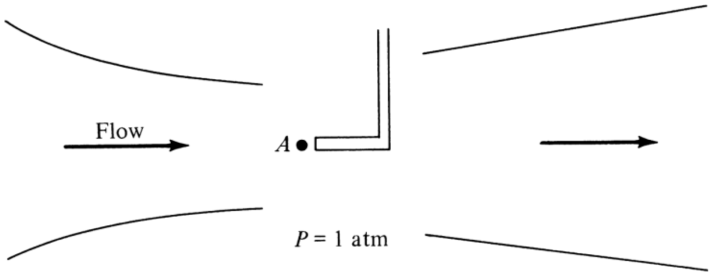
The velocity, or airspeed can now be calculated as:

$$V = \sqrt{\frac{2(p_{tot} - p_s)}{\rho}}$$

This can be applied in a wind tunnel,



(a) Closed test section



(b) Open test section

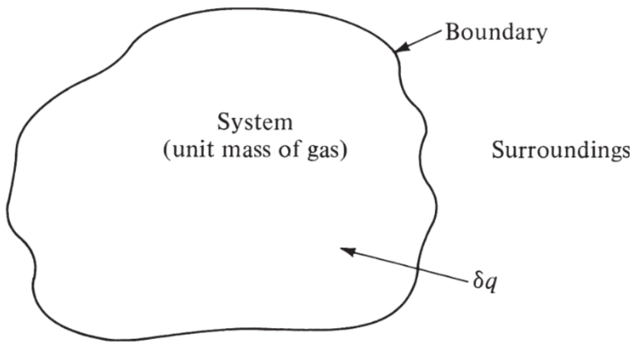
At point B of the pitot probe, the total pressure is obtained. And at point A, the static pressure is obtained. For subsonic wind tunnels, it is fairly safe to assume density constant across the wind tunnel.

The true airspeed is the airspeed in which the change in density is taken into consideration:

$$V_{true} = \sqrt{\frac{2(p_{tot} - p_s)}{\rho}}$$

Lecture 3 Aerothermodynamics and Compressibility

The first law of thermodynamics states that within a system of (for example) gases enclosed by a boundary, the internal energy (intermolecular energy within the system) can be increased by adding heat to the system from the surroundings, or performing work on/by the system. Hence, the change in internal energy is

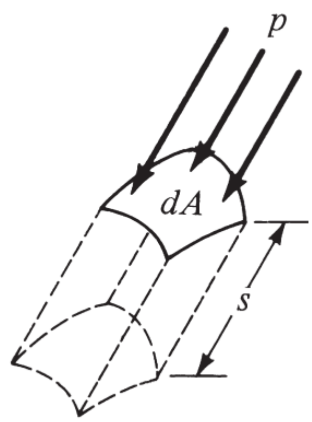


$$\boldsymbol{de = \delta W + \delta q}$$

It is an energy equation stating that the change in internal energy is equal to the sum of the heat added to and the work done on the system.

To get more practical expressions, assume work is done on the system at the boundary layer due to a surrounding pressure,

$$\delta W = \int p dA\, s = p \int s dA$$



$\int sdA$ is the change in volume due to the boundary layer being pushed in, so $\int sdA = -dv$.

$$\delta W = -pdv$$

$$\boldsymbol{de = \delta q - pdv}$$

Assume enthalpy h as the internal energy + the work done on the boundary, then

$$h = e + pv$$

$$dh = de + pdv + vdp$$

This means that

$$\boldsymbol{\delta q = dh - vdp}$$

With these expressions, when heat is added or work is done, the internal properties of the system change (p, ρ, T), this is called a process. In a process, certain variables change and others stay constant. When the volume is constant, it is a constant volume process and other variables change.

Specific heat is defined as the amount of heat added to increase the temperature by 1 K. Since the change in temperature is different for certain processes, there are two specific heats

$$c_v = \left(\frac{\delta q}{dT}\right)_{constant\ vol}$$

$$c_p = \left(\frac{\delta q}{dT}\right)_{constant\ pres}$$

These are treated as constants for a specific type of gas. Using the variations of the 1st law of thermodynamics,

$$de = c_v dT$$
$$dh = c_p dT$$
$$e = c_v T$$
$$h = c_p T$$

The equations above are derived from,

- For constant volume, $de = \delta q$
- For constant pressure, $\delta q = dh$

Isentropic flow is both adiabatic ($\delta q = 0$) and reversible (no frictional forces). This doesn't mean that temperature and density do not vary. When the volume varies, work is done, hence the internal energy changes, and hence the temperature changes. This argument holds for compressible flows, where the density is variable.

To get the temperature and density variation,

$$de = -pdv = c_v dT$$

$$dh = c_p dT = vdp$$

$$\frac{-pdv}{c_v} = \frac{vdp}{c_p} \rightarrow \frac{dp}{p} = -\frac{c_p}{c_v}\frac{dv}{v} \rightarrow \frac{dp}{p} = -\gamma\frac{dv}{v}$$

$\gamma = 1.4 = \frac{c_p}{c_v} = const$

Integrating this will result in, using $p = \frac{m}{v}RT$ and,

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \quad \text{isentropic flow}$$

This holds for the same streamline, but also if all flows are uniform, they hold for any point (so it can be compared to sea level conditions if an airplane is flying at that altitude).

Now we want to relate temperature and velocity with each other.

Considering isentropic flow,

$$dh - vdp = 0 \rightarrow dh + v\rho VdV = 0$$

Considering flow of unit mass, $m = \rho v = 1\ g$.

$$dh = -VdV$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h + \frac{V^2}{2} = \text{const}$$

Note that $h = c_p T$,

$$c_p T_1 + \tfrac{1}{2}V_1^2 = c_p T_2 + \tfrac{1}{2}V_2^2$$

$$c_p T + \tfrac{1}{2}V^2 = \text{const}$$

To summarize,

- For incompressible flow

$$A_1V_1 = A_2V_2 \qquad \text{continuity}$$

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \qquad \text{Bernoulli's equation}$$

- For isentropic compressible flow,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad \text{continuity}$$

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma/\gamma-1} \qquad \text{isentropic relations}$$

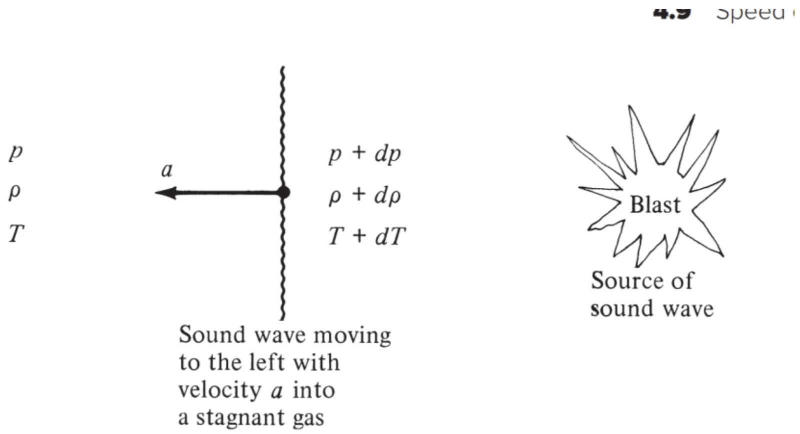
$$c_pT_1 + \tfrac{1}{2}V_1^2 = c_pT_2 + \tfrac{1}{2}V_2^2 \qquad \text{energy}$$

$$p_1 = \rho_1 RT_1$$

$$p_2 = \rho_2 RT_2 \qquad \text{equation of state}$$

Lecture 4 Speed of Sound

Sound waves travel through the air at the speed of sound. Consider a firecracker that creates a sound wave. The sound moves with a velocity a to the left. The sound wave itself is a thin region of disturbance in the air, across which the pressure, temperature and density change slightly (this change in pressure activates you eardrum). Behind the wave, the pressure, density, and temperature have changed due to the sound wave.



Now imagine you are sitting on the moving wave. The sound seems to stand still from your vantage point. The air in front of the wave appears to be coming at you with velocity a. As it passes through the sound wave, the pressure, temperature, and density of the air are slightly changed by the amounts $dp, dT, d\rho$. The airspeed then changes slightly as well by da. Both scenarios are equivalent, one in which you stand and watch the sound wave come by, and the other one riding on top of the wave and watching the air go by.

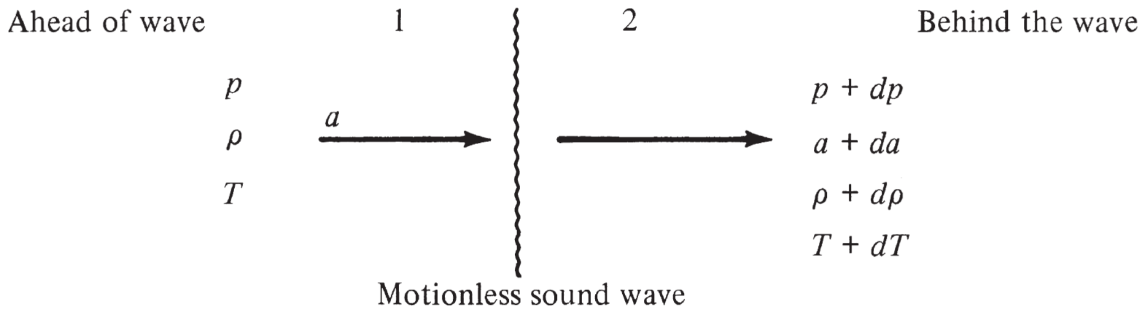


Figure 4.13 Model with the sound wave stationary.

Applying the continuity equation for points 1 and 2:

$$\rho A_1 a = (\rho + d\rho)A_2(a + da)$$

In the figure, the area stays constant, so

$$\rho a = (\rho + d\rho)(a + da)$$

$$\rho a = \rho a + a d\rho + \rho da + d\rho da$$

The product $d\rho da$ is so small that it can be neglected. Solving for a,

$$a = -\rho \frac{da}{d\rho}$$

Applying Euler's equation

$$dp = -\rho a da \quad \rightarrow \quad da = -\frac{dp}{\rho a}$$

Substituting both expressions:

$$a^2 = \frac{dp}{d\rho}$$

The flow through a sound wave involves no heat addition, hence it can be considered isentropic.

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)}$$

Hence

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho}$$

Therefore,

$$a = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma R T}$$

This means that **the speed of sound in a perfect gas depends only on the temperature of the gas.**

The Mach number is ratio between the airspeed and speed of sound:

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma R T}}$$

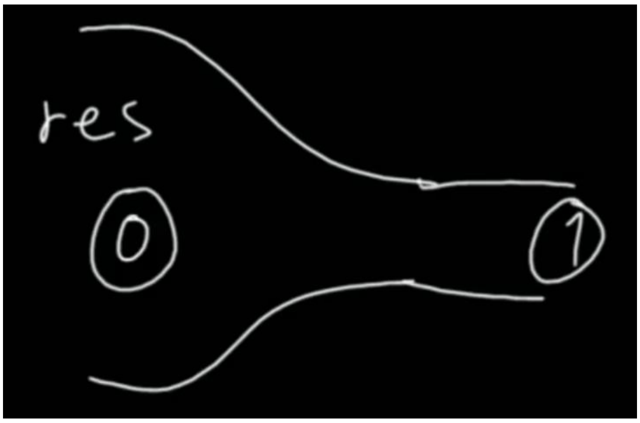
We can use the Mach number to classify different types of flight:

- Subsonic: $M < 0.8$
- Sonic : $M = 1$
- Supersonic: $M > 1$.

At low-speed subsonic wind tunnels where $M < 0.3$, Bernoulli's equation were applied to calculate the velocity:

$$V = \sqrt{\frac{2(p_t - p_s)}{\rho}}$$

However, for high-speed flows, compressibility must be taken into account.



We can apply the energy equation, knowing that the velocity at the reservoir is zero:

$$\boldsymbol{\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2}$$

Knowing that

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \quad \text{isentropic flow}$$

We end up with:

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{\gamma - 1}}$$

These expressions can be used to calculate the pressure, temperature, and density of a point, if u compare it with a point where the velocity is zero, such as in a reservoir. This is the second relation of isentropic flow.

We can use this to determine whether a flow is considered compressible or not. Knowing that at $M < 0.3$, the flow can be considered incompressible. So

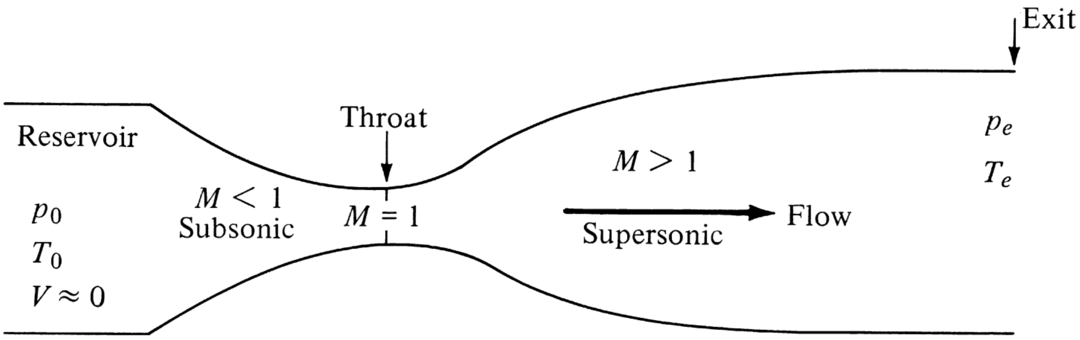
$$\frac{\rho_1}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} 0.3^2\right)^{\gamma - 1} = 0.05$$

Hence, if the difference in density is less than 5%, the flow is incompressible.

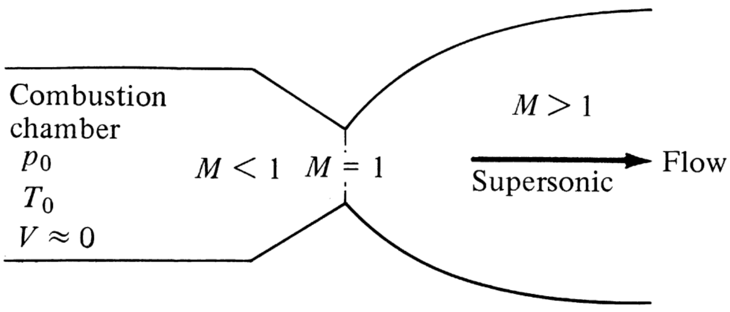
Supersonic Nozzles

We have a **reservoir** with air under high pressure and temperature. When we open the valve, the air will speed up and flow through a converging channel until it reaches the point with the smallest area, the **throat**, where the Mach number is 1.

In order to get supersonic flow, after the throat the area increases. You might think that we need to further the decrease the area. This will be debunked.



(*a*) Supersonic wind tunnel nozzle



(*b*) Rocket engine nozzle

Recall the continuity equation:

$$\ln(\rho) + \ln A + \ln V = \ln(cons)$$

Differentiating

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

Applying Euler's equation,

$$\rho = -\frac{dp}{VdV}$$

to the previous one:

$$-\frac{d\rho\, VdV}{dp} + \frac{dA}{A} + \frac{dV}{V} = 0$$

Recall that for isentropic flow,

$$a^2 = \frac{dp}{d\rho} \rightarrow \frac{d\rho}{dp} = \frac{1}{a^2}$$

Substituing:

$$-\frac{VdV}{a^2} + \frac{dA}{A} + \frac{dV}{V} = 0$$

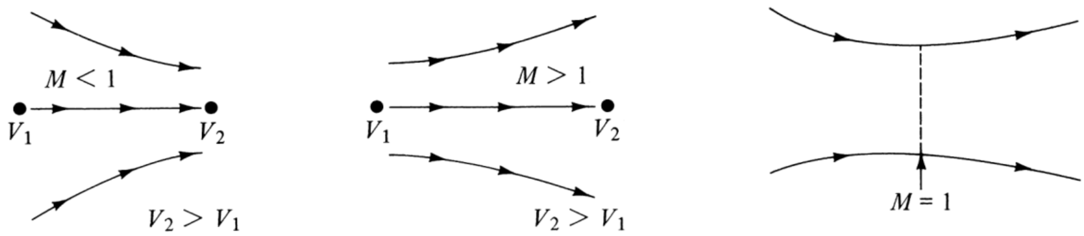
Rearranging we get,

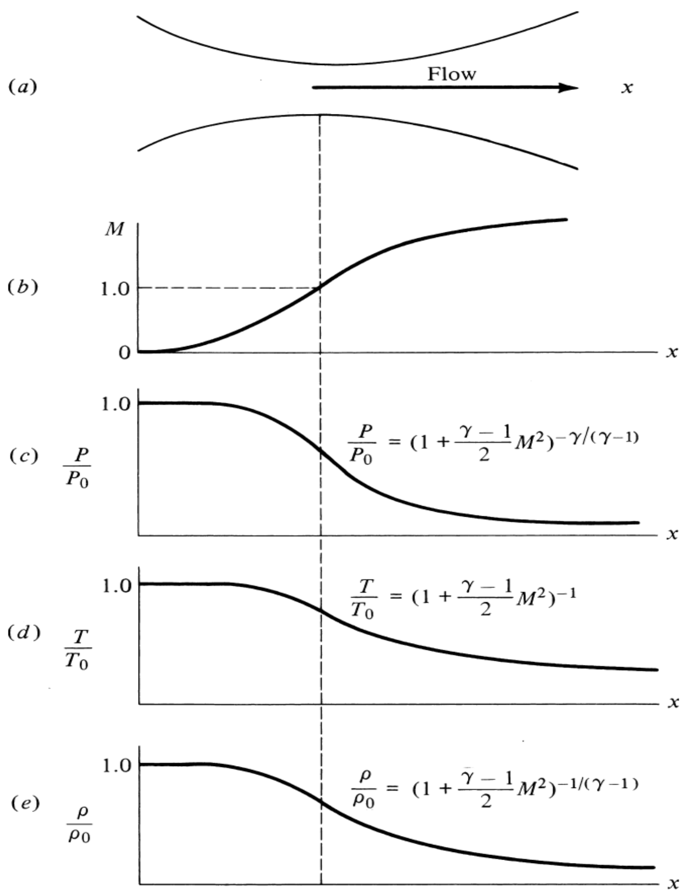
$$\boldsymbol{\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}}$$

This is the ***area-velocity relation***.

We can find some information:

1. If the flow is subsonic $M < 1$, then for the velocity to increase ($dV > 0$), the area must decrease, because the expression on the left is negative. $\frac{dA}{dV} = negative$
2. If the flow is supersonic $M > 1$, then for the velocity to increase, the area must also increase, $\frac{dA}{dV} = positive$.
3. If the flow is sonic, $M = 1$: $\frac{dV}{V} = \frac{dA}{A}\frac{1}{0}$. This is impossible, thus the only to have a finite $\frac{dV}{V}$ is if $\frac{dA}{A} = 0 \rightarrow \frac{dV}{V} = \frac{0}{0} = finite\ number$. Hence, the area must be minimum, which occurs at the throat.





To summarize what happens to the airflow at a supersonic nozzle, consider the following graph.

1. The mach number increases over the nozzle.
2. Applying the equation, the corresponding pressure, temperature, and density all decrease.

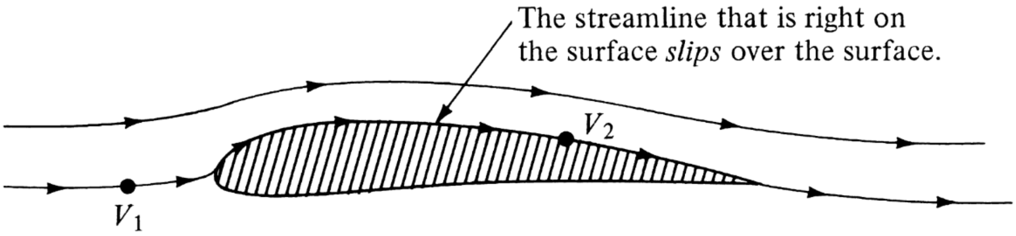
Lecture 5 Viscous flows

Up until now, all flows, including isentropic, was assumed to be inviscid (no friction). This section is about the effect of friction. Pressure is the result of molecular impacts perpendicular to the surface, whilst friction creates a shear stress tangentially.

The flow on the left is assumed to have no friction: the pressure distribution over the forward surface exactly balances that over the rear surface, hence there is no drag (aerodynamic force). On the right side, the flow separates at the rear surface: setting up a complicated flow and a pressure distribution that is less on the rear surface than on the forward surface, hence a force called drag.

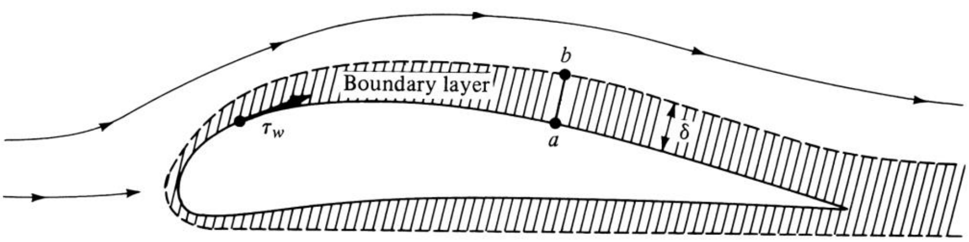
Frictionless flow: no drag   Real flow: finite drag

The airfoil sketch below is assumed to have no friction, where the streamline right at the surface slips over the surface. Bernoulli's equation can be used to calculate V_1 and V_2.

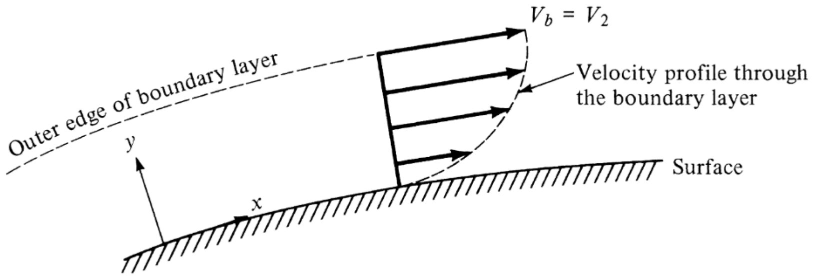


However, ***the flow at the surface adheres (sticks) to the surface*** because of friction between the gas and the solid material. Here, there is a thin region of retarded (delayed) flow due to friction: **boundary layer.** The inner edge of the boundary layer is the solid surface where $V = 0$. At the outer edge of the boundary layer, point b, the velocity is equal to V_2 from the frictionless model, using Bernoulli's equation. Hence, on an airfoil, a flow field can be split into a boundary layer, where friction is important, and another region of frictionless flow.

4.15 Introduction to Viscous Flow



The static pressure within the boundary layer, p_a, p_b will stay constant. That is because friction is acted tangentially, while the static pressure is a perpendicular pressure distribution. Let δ be the boundary layer thickness which will grow as the flow moves over the body. In addition, the friction creates a shear stress τ_w tangentially to the surface.

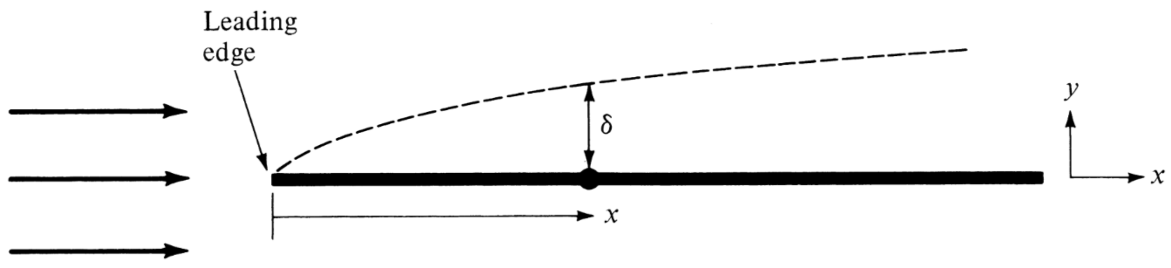


A velocity profile is sketched above. The velocity starts at zero at the surface and increases continuously until V_2. V is then a function of y. At the surface where $y = 0$, $\left(\frac{dV}{dy}\right)_{y=0}$ can be used for the shear stress:

$$\tau_W = \mu\left(\frac{dV}{dy}\right)_{y=0}$$

μ is viscosity coefficient, different for gases and liquids in mass/length/time. For a gas, μ increases as T increases. For liquids, μ decreases as T increases. For air, $\mu = 1.7894 \cdot 10^{-5}\frac{kg}{ms}$.

Let x be distance from the leading edge to trailing edge. V_∞ is the free-stream velocity far-outside the plate.

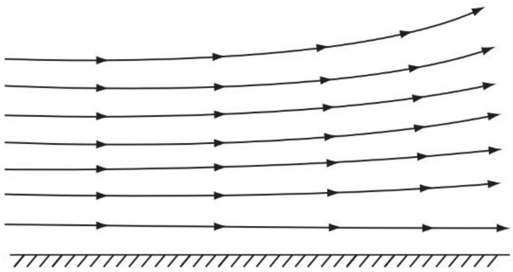


Reynolds’ number is defined as:

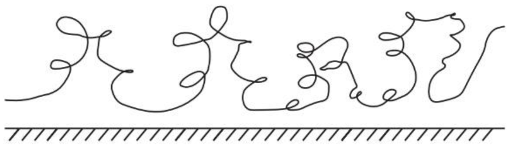
$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

There are two types of viscous flow:

- Laminar flow: in which the streamlines are smooth and regular and a fluid element moves smoothly along a streamline.
- Turbulent flow: in which the streamlines break up and a fluid element moves in a random, irregular fashion.



(a) Laminar flow



(b) Turbulent flow

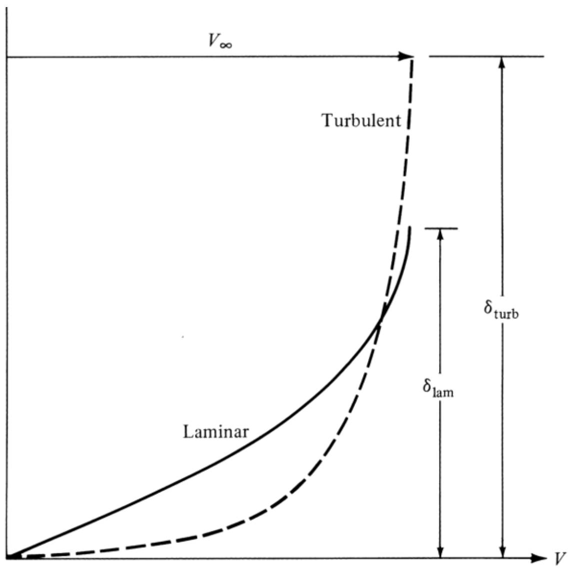
When making a velocity profile, you can see that the velocity changes much faster over the boundary layer y at turbulent flow than laminar flow:

$$\left(\frac{dV}{dy}\right)_{y=0,turbulent} > \left(\frac{dV}{dy}\right)_{y=0,laminar}$$

This means that

$$\tau_{W_{turbulent}} > \tau_{W_{laminar}}$$

Most flows in aerodynamics, are turbulent and therefore skin friction also considers turbulent flow. That is unfortunate, but most slender wings are designed to encourage laminar flow.



Laminar flow

To calculate the laminar boundary layer thickness, we can use the approximation obtained from experiment:

$$\boldsymbol{\delta = \frac{5.2x}{\sqrt{Re_x}}\ laminar}$$

A flow as complex as laminar can therefore be expressed by a very simple equation. Note that because $Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$, $\delta \propto x^{\frac{1}{2}}$. As for the shear stress, it is more convenient to introduce the **local skin friction coefficient:**

$$c_{f_x} = \frac{\tau_w}{q_\infty}$$

For laminar flow:

$$\boldsymbol{c_{f_x} = \frac{0.664}{\sqrt{Re_x}}}$$

Combining both equation, the shear stress as a function of x becomes:

$$\boldsymbol{\tau_w = \frac{0.664 q_\infty}{\sqrt{Re_x}}}$$

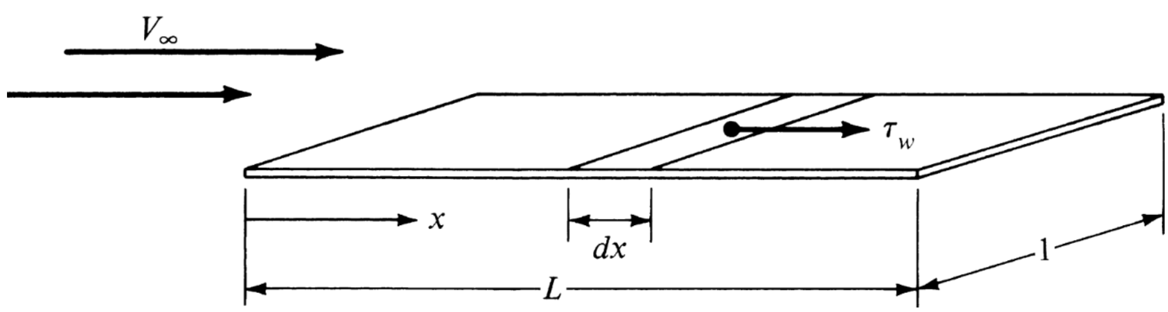
Note that c_{f_x} and $\tau_w \propto \frac{1}{\sqrt{x}}$.

In conclusion, there is a shear stress distribution along the surface of the airfoil. The total **skin friction drag $\boldsymbol{D_f}$** for laminar flow becomes:

$$D_f = \int_{LE}^{TE} \tau_w dx = \int_0^L \tau_w dx = \int_0^L \frac{0.664 q_\infty}{\sqrt{Re_x}} dx = \frac{0.664 q_\infty}{\sqrt{\frac{\rho_\infty V_\infty}{\mu_\infty}}} \int_0^L \frac{dx}{\sqrt{x}} dx = \frac{1.328 q_\infty L}{\sqrt{\frac{\rho_\infty V_\infty L}{\mu}}}$$

$$\boldsymbol{D_f = \frac{1.328 q_\infty L}{\sqrt{\frac{\rho_\infty V_\infty L}{\mu}}}}$$

Let C_f be the **skin friction coefficient of the entire plate:**



$$C_f = \frac{D_f}{q_\infty S}$$

Knowing that $S = L$, for laminar flow,

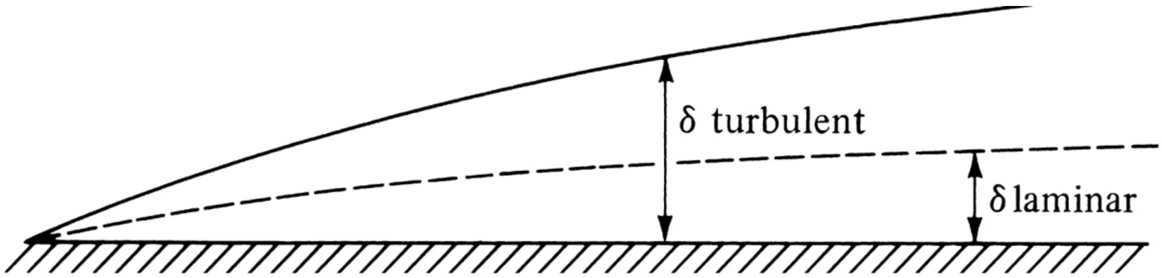
$$C_f = \frac{1.328 q_\infty L}{\sqrt{\frac{\rho_\infty V_\infty L}{\mu}}} \cdot \frac{1}{q_\infty L}$$

$$\boldsymbol{C_f = \frac{1.328}{\sqrt{Re_L}}}$$

Where Re_L is the Reynold's number for the entire plate, while Re_x is a function of x. The number shown above only applies for low-speed, subsonic flow. Analytically, you could calculate these values, but beyond the scope of this paper.

Turbulent flow

The turbulent boundary layer will be thicker than a laminar boundary layer. Unlike laminar flow, there are no analytical ways to calculate the boundary layer, but there is a good approximation



That is way we only use experimental result.

$$\boldsymbol{\delta = \frac{0.37x}{Re_x^{0.2}}\ turbulent}$$

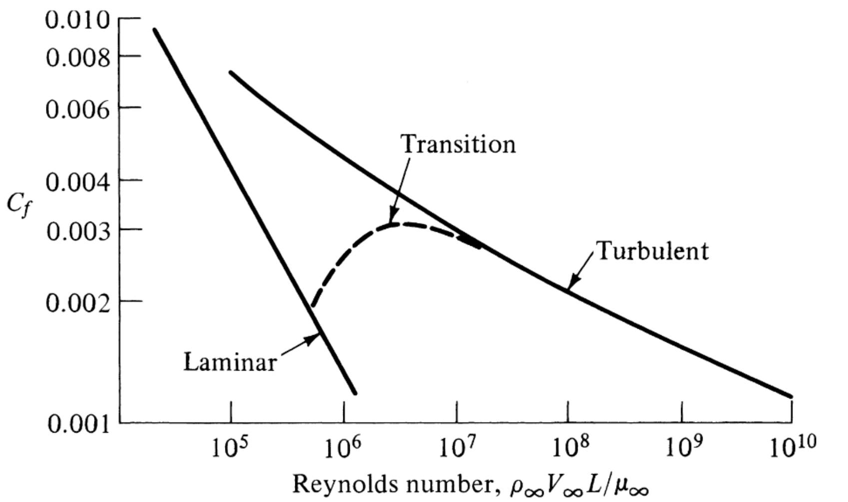
From this, turbulent boundary $\delta \propto x^{4/5}$. This means that turbulent boundary layers grow faster and are thicker than laminar. As for the local skin friction coefficient:

$$\boldsymbol{c_{f_x} = \frac{0.0592}{(Re_x)^{0.2}}\ turbulent}$$

Using the same previous method,

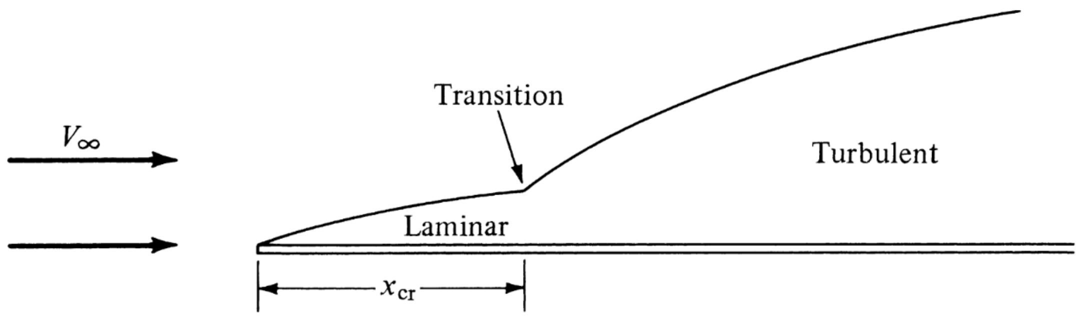
$$\boldsymbol{C_f = \frac{0.074}{Re_L^{0.2}}\ turbulent \rightarrow D_{sk} = \frac{1}{2} C_f \rho V^2 S}$$

Note that $C_f \propto L^{-1/5}$ for turbulent, $C_f \propto L^{-1/2}$ for laminar. These differences are shown:



Transition

In an airfoil, the flow always starts out from the leading edge as laminar. Then, at some point downstream of the leading edge, the laminar boundary layer becomes unstable and turbulent flow begin to grow by $x^{\frac{4}{5}}$



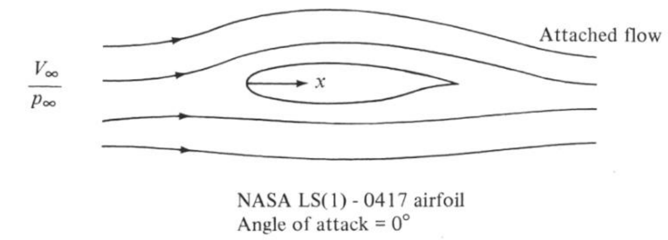
Let x_{cr} be the transition point or the critical value. The local Reynolds number is the **critical Reynolds number**

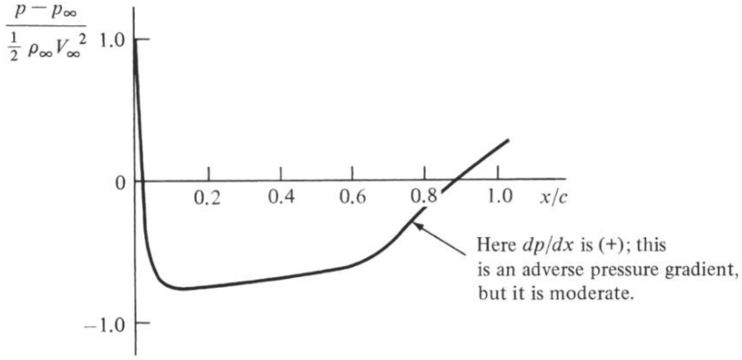
$$Re_{x_{cr}} = \frac{\rho V x_{cr}}{\mu}$$

The position of the transition depends on the Mach number (airspeed), temperature (viscosity coefficient) and altitude (density). Usually, the critical Reynolds number is given.

Flow separation

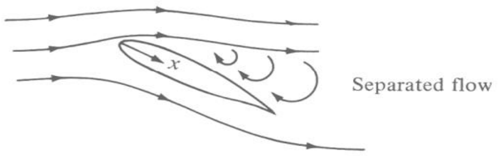
In addition to a shear stress, viscous flows also causes a phenomenon called flow separation, which, in turn, creates an aerodynamic force: **pressure drag due to separation.**



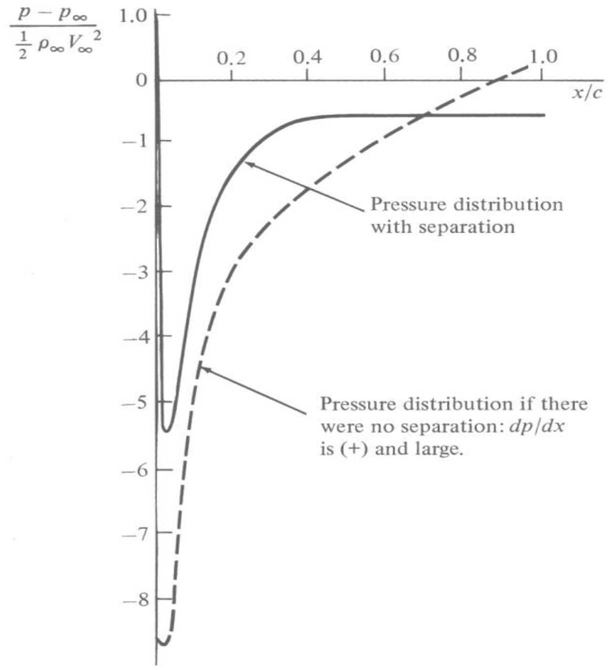


That the lead edge, there is a stagnation region, where the pressure is maximum. When flow the expands around the top surface, the surface pressure decreases dramatically, where the surface pressure is less than the static pressure. Then the pressure increases slightly again, this is the **adverse pressure gradient** $\left(\frac{dp}{dx}\right) > \mathbf{0},$ but still relatively small. This flow does not separate. Now consider the airfoil with a high AoA.

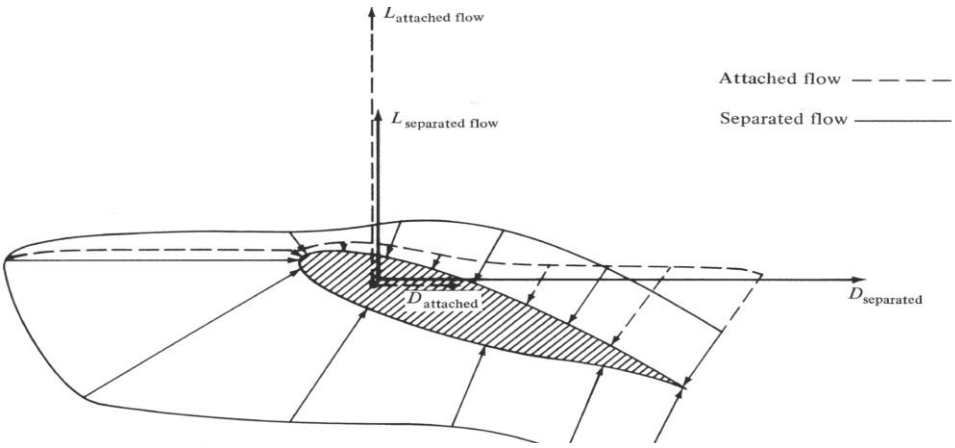
4.56. First assume that we had some magic fluid that would



NASA LS(1) - 0417 airfoil
Angle of attack = 18.4°



The dashed line is for normal conditions: without flow separation. The pressure distribution would follow the dashed line if there was no separation. First, the pressure would drop dramatically downstream of the leading edge far below the static pressure, because the velocity is there the highest (continuity). Then there is an adverse gradient. But afterwards the flow will go slightly above the static pressure at the trailing edge. Here, $\frac{dp}{dx} \gg 0$. If the pressure increases dramatically over the distance, then the real flow tends to separate from the surface. Here, the flow will not dip as much, but neither will it go above the static pressure. The consequences for the lift generation is.



There are two consequences:

1. Loss of lift: High lift is generated when the pressure on bottom surface is larger and the pressure on the top surface is small. Flow separation causes the upper surface to have a high flow pressure **just downstream of the leading edge,** hence reducing lift. Furthermore, there is geometric effect in which just downstream of the leading edge, the airfoil becomes horizontal approximately, contributing to even higher pressure. Often then, the wing stalls.
2. A major increase in drag: Near the trailing edge of the top surface, the pressure is now smaller than the pressure that would exist if the flow were attached. Due to separation, the attached flow is less resulting in smaller lift force to the left. Drag is a horizontal component of the aerodynamic force. Therefore the pressure exerted on this portion of the surface has a strong component. The pressure that acts to the left, at the upper surface, will counteract the pressure acting on the right at the lower surface.
   But because the pressure at the upper surface is lower at the trailing edge, the net force will shift more to the right, hence causing more aerodynamic drag.

But why does the flow separate?

If the adverse pressure gradient, $\frac{dp}{dx}$ is positive, then the pressure will increase over the boundary layer. This means that the fluid elements have to work their way 'uphill' against an increasing pressure. Consequently, the fluid elements will slow down under the influence of a pressure gradient. Near the surface, where the flow has been retarded by friction forces, the velocity is small (velocity profile), but it still experiences the same pressure gradient that the flow outside the boundary layer experiences, where the velocity is much higher. This means that the flow will stop and go in opposite direction. For a turbulent layer, the velocity profile is higher, hence the flow moves faster. **Laminar boundary layers separate more easily than turbulent boundary layers.**

The total drag due to viscous flow,

$$D_{viscous} = D_{friction} + D_{pressure}$$

For laminar flow, pressure drag is bigger than friction drag.

For turbulent flow, pressure drag is smaller than friction drag.

Therefore, there is no preference for which flow.

At high mach numbers,

$$D_{pressure} = D_{separation} + D_{wave}$$

Lecture airfoil lift and drag

Consider the nomenclature of a wing

5.2 Airfoil Nomenclature

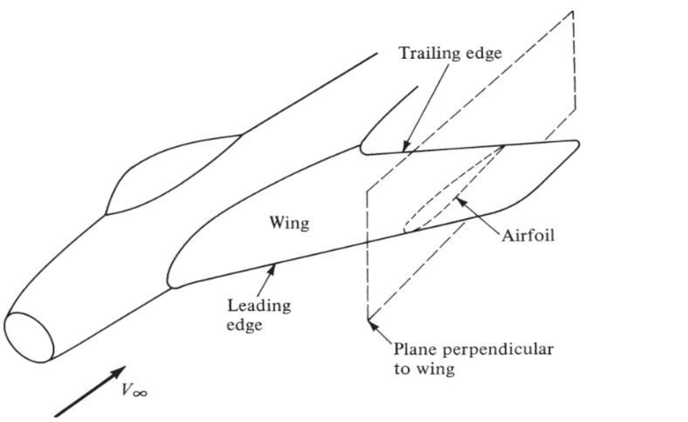
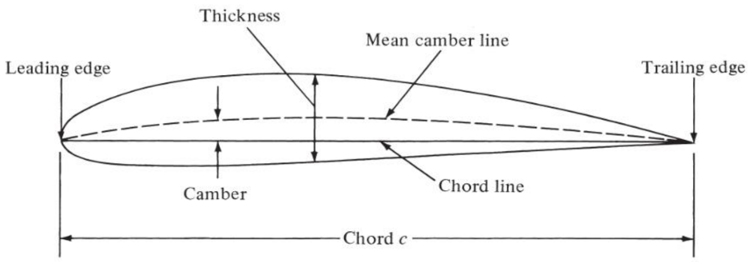


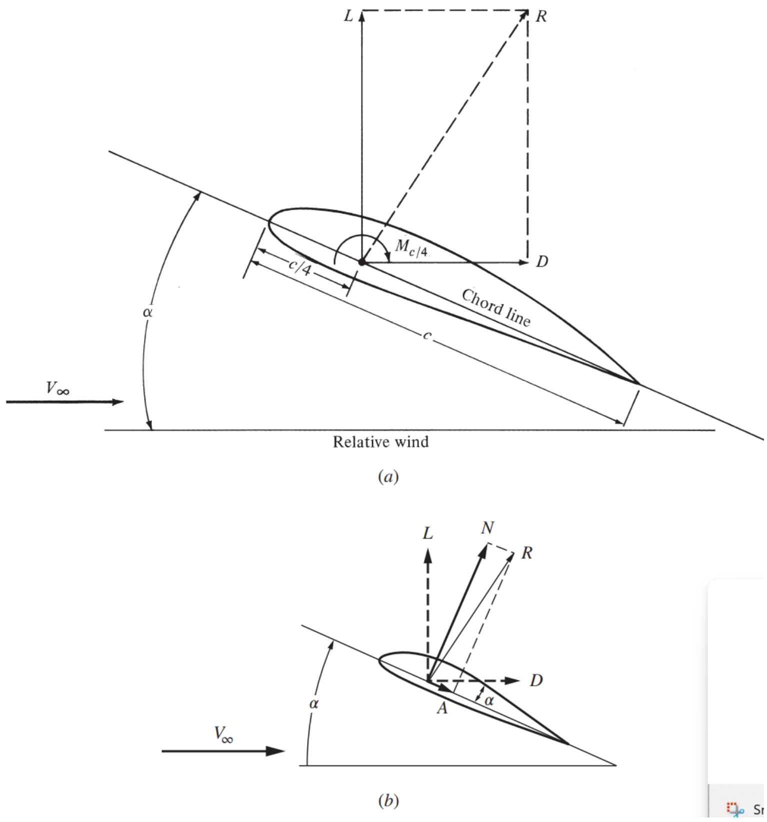
Figure 5.2 Sketch of a wing and airfoil.



- The **mean camber line** which is the locus of points halfway between the upper and lower surfaces.
- The **Leading edge** and **trailing edge** are the most forward and rearward points of the mean camber line.
- The **chord** is the horizontal distance between the leading edge and trailing edge.
- The **camber** is the maximum distance between the mean camber line and the chord.

NACA2415, 2 = maximum camber to chord, position of max. Camber as ratio of chord, 15 = percentage thickness of airfoil.

The camber and thickness of the airfoil control the lift and moment characteristics. Now consider an airfoil with an AoA.



Due to pressure distribution and shear distribution, a net aerodynamic force R is created. L is force perpendicular to R, D is the force parallel to R. In addition, the surface pressure and shear stress distribution create a moment that tends to rotate the wing.

In the section picture, N (normal force) and A (axial force) are the components perpendicular to and parallel to the chord. From this,

$$L = N\cos\alpha - A\sin\alpha$$

$$D = N\sin\alpha + A\cos\alpha$$

These forces are usually not used.

Coefficients

Usually the aerodynamic characteristics depend on:

- Free-stream velocity
- Density
- Wing area
- Angle of attack
- Shape of the airfoil

- Viscosity coefficient (because the aerodynamic forces are generated in part from skin friction distributions)
- The compressibility of the airflow. The Mach number determines the compressibility effects. So speed of sound is also important.

Thus,

$L = f(V_\infty, \rho_\infty, S, \mu_\infty, a_\infty)$

These can be combined into:

$$L = q_\infty S C_L$$

$$D = q_\infty S C_D$$

$$M = q_\infty S C_M c$$

We can rewrite this

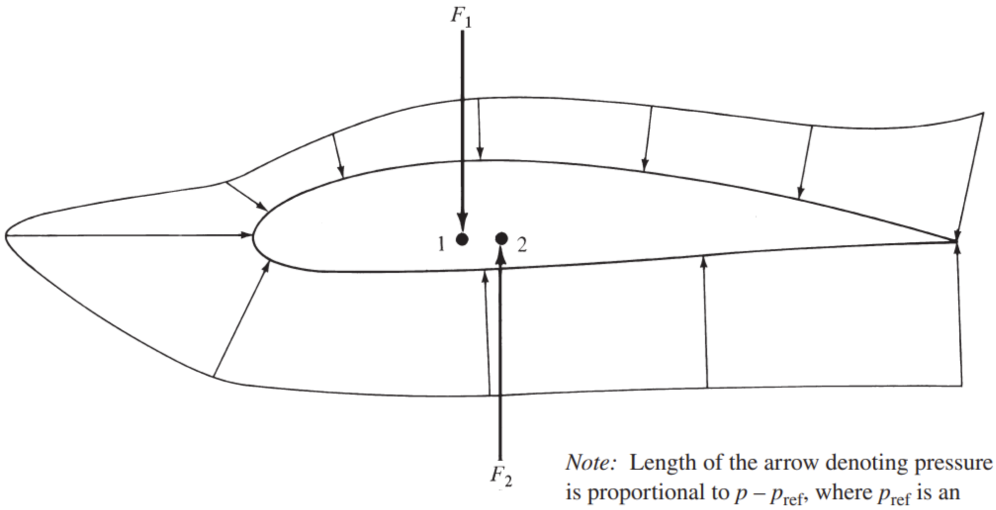
$$C_L = \frac{L}{q_\infty S}$$

$$C_D = \frac{L}{q_\infty S}$$

So the lift coefficient is defined as the aerodynamic lift divided by the dynamic pressure multiplied by a reference area. Note that

$$C_L = f(\alpha, M_\infty, Re)$$

The Mach number (indication of airspeed) and Reynolds number (skin friction). The Mach number can be used to calculate temperature and velocity at different points on the airfoil (isentropic). Both are dependent solely on the free-stream velocity for specific conditions.



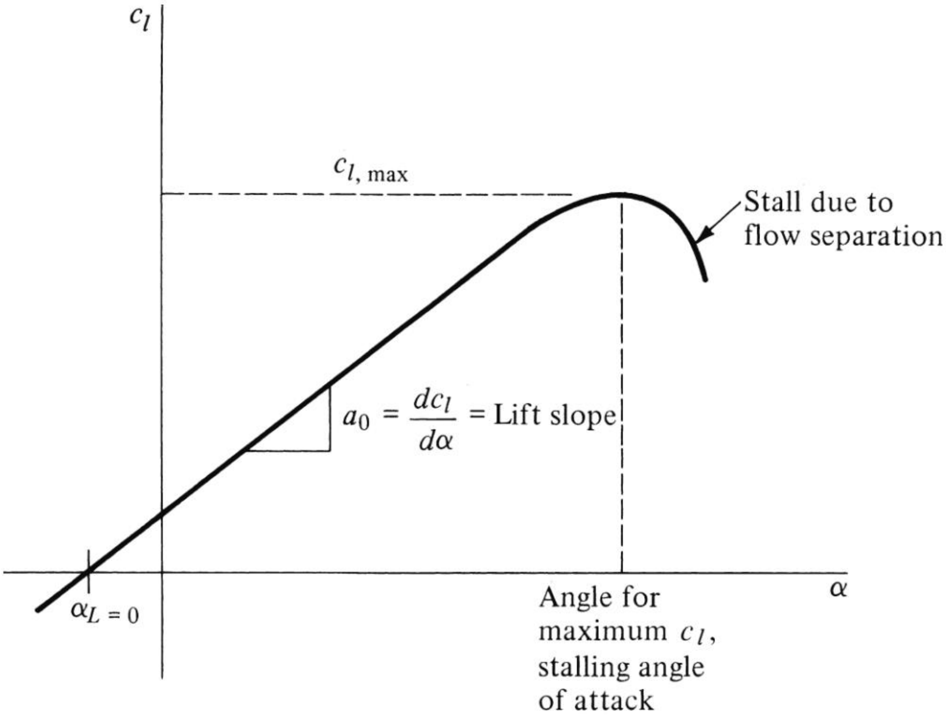
Note: Length of the arrow denoting pressure is proportional to $p - p_{\text{ref}}$, where p_{ref} is an arbitrary reference pressure slightly less than the minimum pressure on the airfoil.

Consider F_1 the force on the airfoil that is created by the pressure distribution at the upper surface and F_2 at the lower surface. $F_R = F_2 - F_1 > 0$, This aerodynamic force will create a moment. The value of this moment depends on the point about which we choose to take moment. Usually, we choose $\frac{x}{c} = 0.25$, $M_{\frac{x}{c}=0.25}$. The moment around a point is different and unique and changes over AoA. However, the $M_{\frac{x}{c}=0.25}$, or the **moment around the aerodynamic centre** will always stay constant and is thus independent of angle of attack.

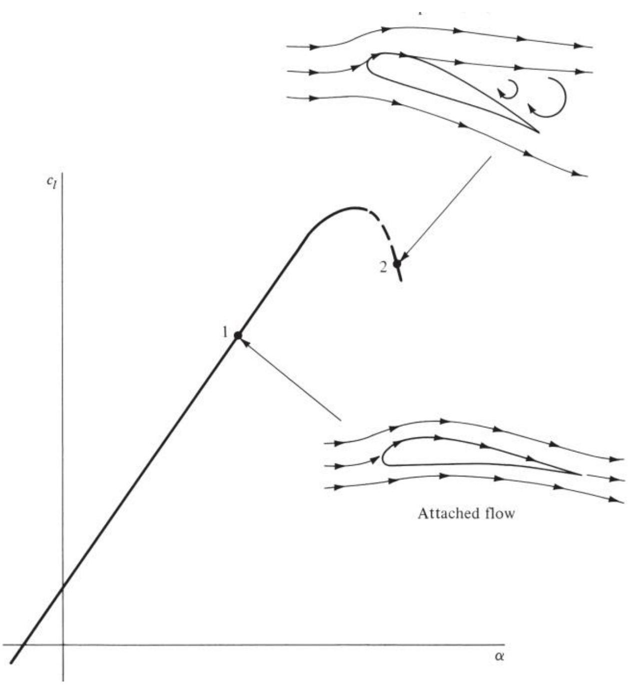
The beauty of these equations are that complex pressure and shear distributions can be replaced by a single dimensionless coefficient.

Airfoil

By changing the angle of attack, the lift coefficient changes linearly for a large while. The camber will cause the airfoil to generate lift even when the AoA=0. For a symmetric airfoil, the zero-lift angle of attack $\alpha_{L=0} = 0$.



At the stalling point the flow starts to separate, hence causing a major decrease in pressure.



The lift per unit span becomes:

$$L = q_\infty c c_l$$

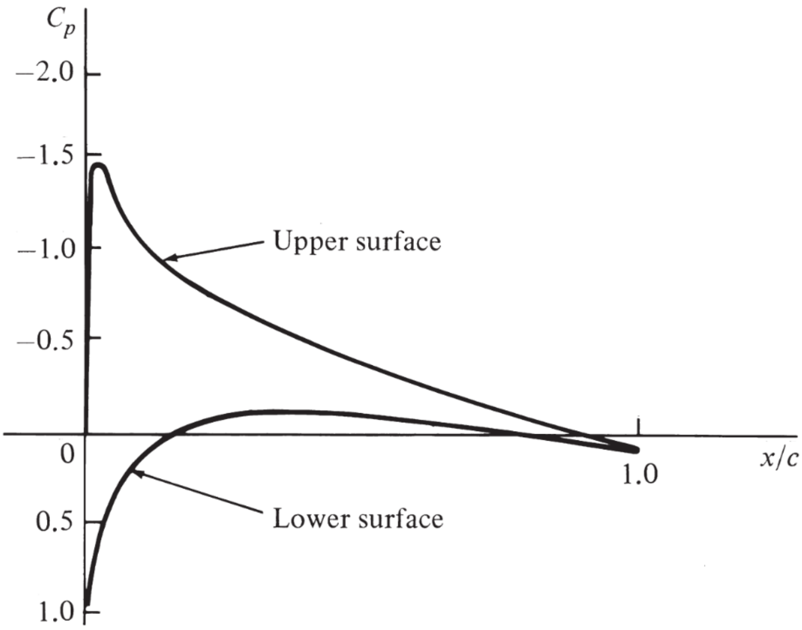
These curves depend on the type of airfoil and wing area etc. these results obtained considered incompressible flow.

Pressure coefficient

If we want to make a pressure distribution, instead of plotting the actual pressure, we can define the pressure coefficient:

$$C_p = \frac{p - p_\infty}{q_\infty}$$

When making a pressure coefficient distribution, the following can be seen. The y-axis is reversed to make it pleasant to read. The pressure decreases dramatically, hence $C_P < 0$.

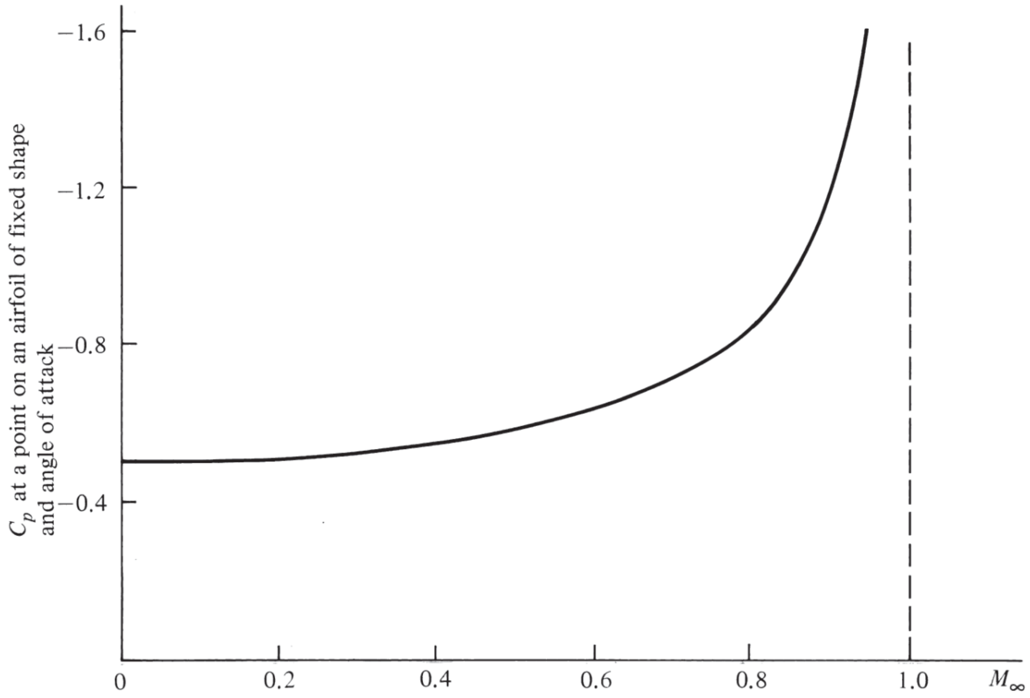


Now consider a wind tunnel. At $M < 0.3$, the flow is considered incompressible. This means that density does not change and the effect of changing the free-stream velocity within the low-Mach number region will be negligible. We call this pressure coefficient at low mach number: C_{P_0}. At $M > 0.3$, compressibility causes change in density, hence hugely changing the pressure coefficient, as can be seen by

We can now express the pressure coefficient in terms of Mach number:

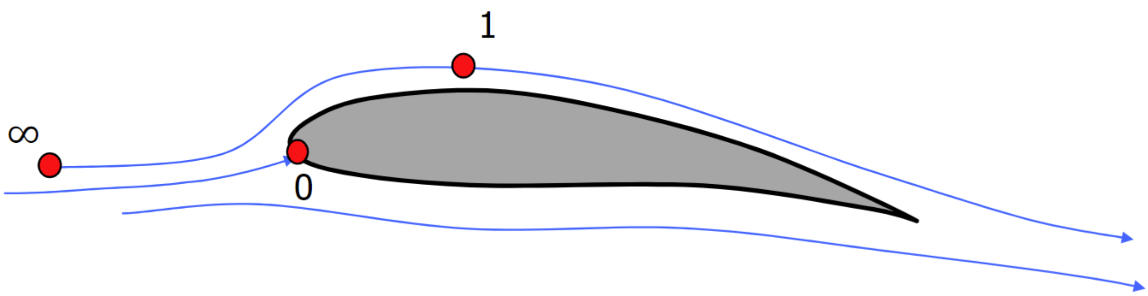
$$C_P = \frac{C_{P_0}}{\sqrt{1 - M^2}}$$

This relation predicts C_p to be infinite at M = 1.



C_{P_0} is the pressure coefficient at $0 < M < 0.3$

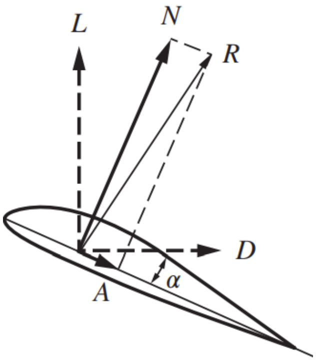
We can combine Bernoulli's equation and the pressure coefficient for low Mach number

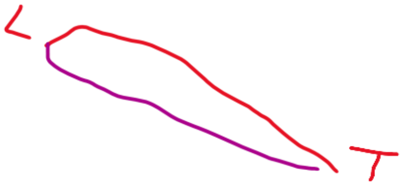


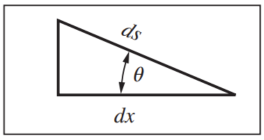
$$C_{P_1} = \frac{p_1 - p_\infty}{q_\infty} = \frac{\frac{1}{2}\rho(V_1^2 - V_\infty^2)}{q} = 1 - \left(\frac{V_1}{V_\infty}\right)^2$$

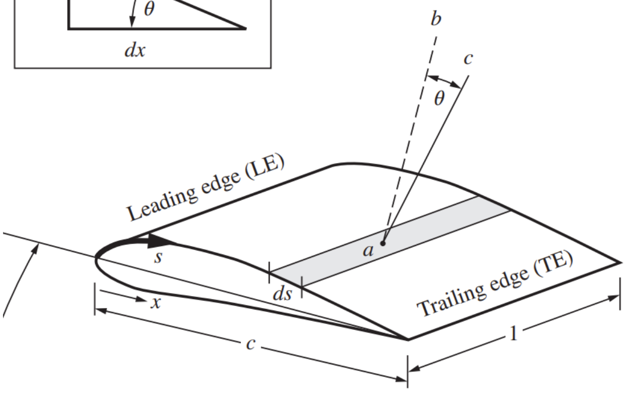
The pressure distribution over an airfoil at upper and lower surface can be obtained from wind tunnel measurements using, for example, a manometer.

Consider the following pictures.









N is the force perpendicular to the chord line, A is the force parallel. s is the distance covered from LE to TE. x is the horizontal distance covered from LE, where $0 < x < chord\ length$. The length of the airfoil is 1 m. Line ab is perpendicular to chord length (in direction N), ac is perpendicular to the airfoil area (in direction of aerodynamic force). The aerodynamic force acting downwards on the infinitesimal gray shaded area is $-pds$. In the direction normal to chord line (N), $-p_{upper}\cos\theta\, ds$, along the entire upper part:

$$\int_{LE}^{TE} -p_{upper}\cos\theta\, ds$$

For the lower part,

$$\int_{LE}^{TE} p_{lower}\cos\theta\, ds$$

Total normal force is:

$$N = \int_{LE}^{TE} p_{lower}\cos\theta\, ds - \int_{LE}^{TE} p_{upper}\cos\theta\, ds$$

Since $dx = ds\cos\theta$,

$$N = \int_0^c p_{lower} dx - \int_0^c p_{upper} dx = \int_0^c p_{lower} - p_\infty dx - \int_0^c p_{upper} - p_\infty dx$$

We can turn this into a coefficient C_N,

$$C_N = \frac{N}{q_\infty S} = \frac{N}{q_\infty c} = \frac{1}{c}\int_0^c \frac{p_{lower} - p_\infty}{q_\infty} dx - \int_0^c \frac{p_{upper} - p_\infty}{q_\infty} dx$$

$$\frac{p_{lower} - p_\infty}{q_\infty} = C_{p,lower}, \frac{p_{upper} - p_\infty}{q_\infty} = C_{p,upper}$$

$$C_N = \frac{1}{c}\int_0^c C_{p,lower} - C_{p,upper} dx = \int_0^1 C_{p,lower} - C_{p,upper} d\frac{x}{c}$$

Since $L = N\cos\alpha + A\ \sin\alpha$, $C_L = C_N\cos\alpha - C_A\sin(\alpha)$. However, for small angle of attacks, the contribution of A to the lift force is small, so therefore, $C_L = C_N$. Usually this is true until $\alpha > 5$.

This means that whenever u obtain a pressure coefficient distribution for upper and lower surface with respect across the chord. The area between the curves is equal to the lift coefficient.

We can also replace the lift coefficient by the compressibility correction:

$$C_L = \frac{C_{L_0}}{\sqrt{1 - M_\infty^2}}$$

This expression may only be used at Mach number larger than 0.3. The $C_L - \alpha$ and the lift coefficient obtained from the pressure distribution curves thus now have all been measured at low speed, so those are the C_{L_0}

Generation of lift

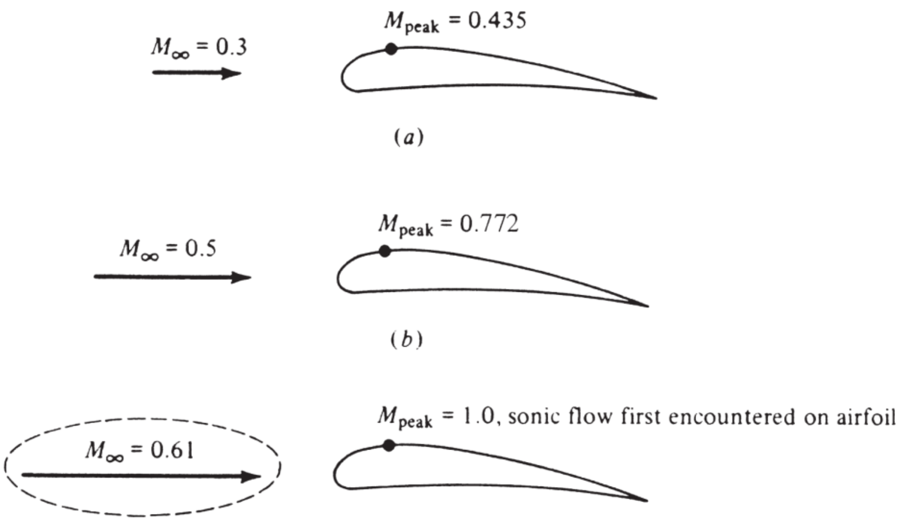
The flow at the upper side of the airfoil is squashed and more dense due to the camber, causing less space for the flow the travel. According to the continuity equation, the flow speed needs to increase. This means that the pressure of the flow will decrease. At the lower surface, the opposite holds, hence the difference in pressure creates an aerodynamic force.

Some alternative explanations:

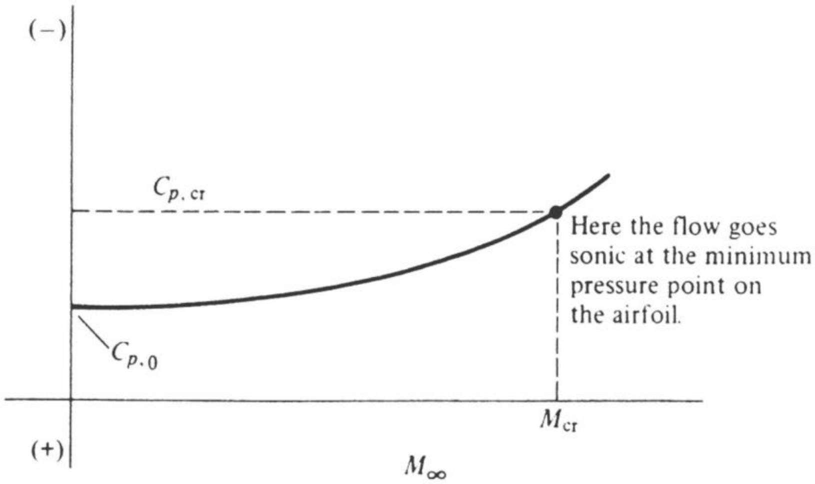
- The airfoil accelerates the air downwards. Based on Newton's third law, the airfoil will feel an opposite force upwards. This is not an explanation but more a consequence.

Lecture critical Mach number

If air flows over an airfoil, the gas expands around top surface and the velocity starts to increase locally. There are certain regions within the airfoil where the local Mach number can be greater than M_{∞}. At one point, there is a free-stream Mach number that causes local flow of air to become sonic $M_{peak} = 1$. The **critical Mach number** $\boldsymbol{M_{cr}}$ is the Mach number at which sonic flow is obtained somewhere on the airfoil.

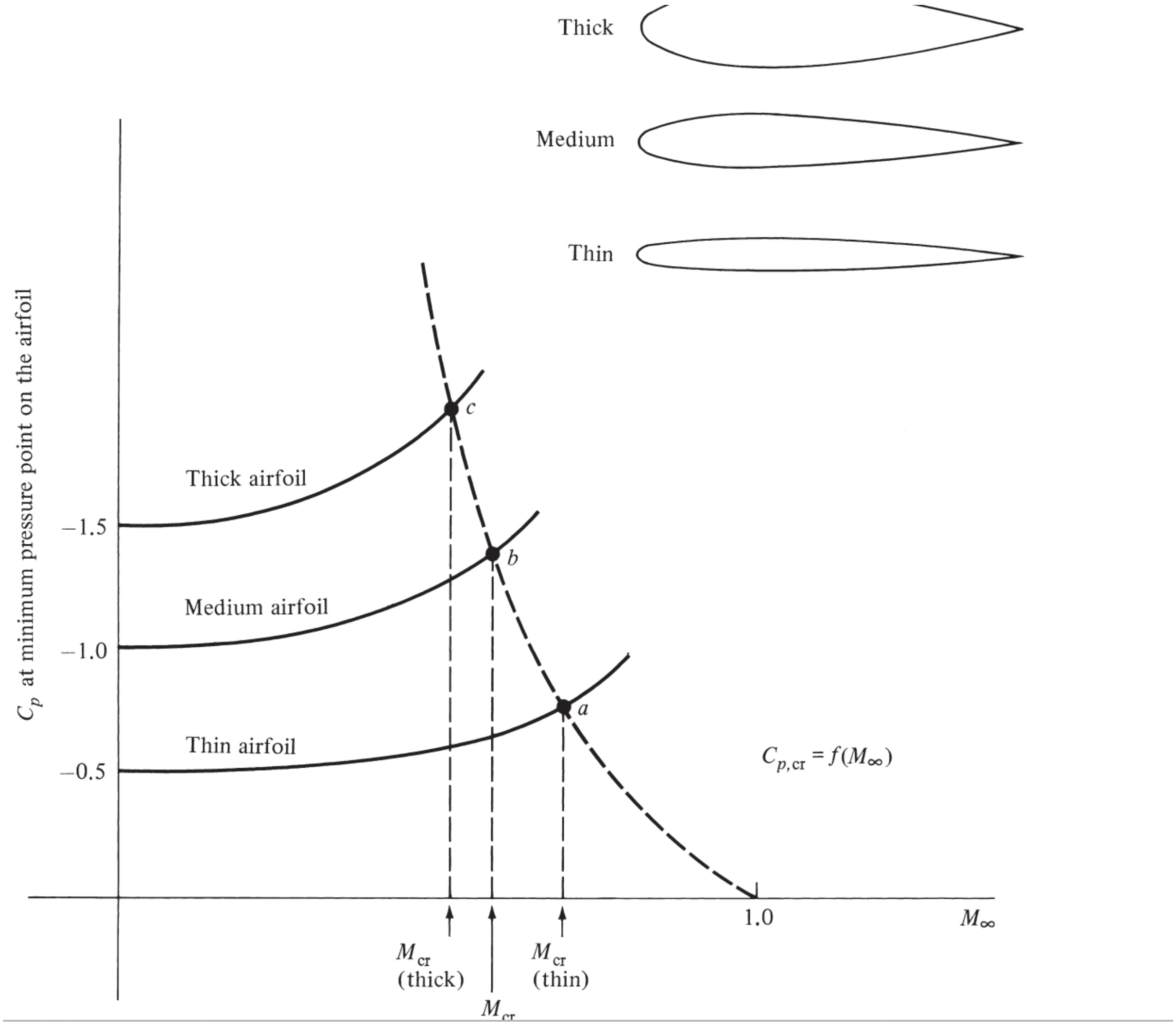


This is an important quantity as sonic flows causes shock waves, thus a lot of drag. The point of Maximum Mach number is also the point where the pressure, and thus, the pressure coefficient is minimum. According to $C_P = \frac{C_{P_0}}{\sqrt{1-M^2}}$, the pressure coefficient will become the most negative. This is the **critical pressure coefficient** $\boldsymbol{C_{P_{cr}}}$.



- A thin airfoil will not curve the flow that much, resulting in small expansion over the top surface. The velocity is low, hence the critical pressure coefficient is small. A small critical pressure coefficient means that the critical Mach number is high. C_{P_0} is small as well as the rate of increase w.r.t the Mach number.
- A thick airfoil causes stronger expansion, causing more decrease in pressure and hence, more pressure coefficient. The increase in pressure coefficient w.r.t the Mach number is big. That means that the critical pressure coefficient is higher and the corresponding Mach number is lower. The local sonic

conditions will be achieved quicker.



You will see that the critical pressure coefficient also decreases with Mach number form a curve. Hence, $C_{p_{cr}} = f(M_\infty)$. We can derive this.

First rewriting the pressure coefficient

$$C_P = \frac{p - p_\infty}{q} = \frac{p_\infty}{q}\left(\frac{p_\infty}{p} - 1\right)$$

Knowing that $q = \frac{1}{2}\rho V_\infty^2$,

$$q = \frac{1}{2}\frac{\rho}{\gamma p_\infty} p_\infty \gamma V_\infty^2 = \frac{\frac{1}{2}V_\infty^2}{\frac{\gamma p_\infty}{\rho}}(\gamma p_\infty)$$

Knowing that $a^2 = \frac{\gamma p_\infty}{\rho_\infty}$

$$q = \frac{1}{2}\frac{V^2}{a^2}\gamma p_\infty = \frac{\gamma}{2}p_\infty M_\infty^2$$

The flow over an airfoil can be considered isentropic so, using

$$\boldsymbol{\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2}M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\boldsymbol{\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

The first equation is the ratio between the total pressure of the flow and the fre-stream pressure, while second equation relates the pressure at any given point to the total pressure. Dividing both to get rid of p_0,

$$\frac{p}{p_\infty} = \left(\frac{1 + \frac{1}{2}(\gamma - 1)M_\infty^2}{1 + \frac{1}{2}(\gamma - 1)M^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Substituting this into the pressure coefficient

$$C_p = \frac{p_\infty}{\frac{\gamma}{2}p_\infty M_\infty^2}\left(\left(\frac{1 + \frac{1}{2}(\gamma - 1)M_\infty^2}{1 + \frac{1}{2}(\gamma - 1)M^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)$$

$$\boldsymbol{C_p = \frac{2}{M_\infty^2}\left(\left(\frac{1 + \frac{1}{2}(\gamma - 1)M_\infty^2}{1 + \frac{1}{2}(\gamma - 1)M^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)}$$

This expression relates the pressure coefficient at any point on the flow field to the free-stream Mach number and the local Mach number at that point. To find the critical pressure coefficient, $M = 1$, $\boldsymbol{M_\infty^2 = M_{cr}^2}$ and then it becomes

$$\boldsymbol{C_{p_{cr}} = \frac{2}{M_{cr}^2}\left(\left(\frac{1 + \frac{1}{2}(\gamma - 1)M_{cr}^2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)}$$

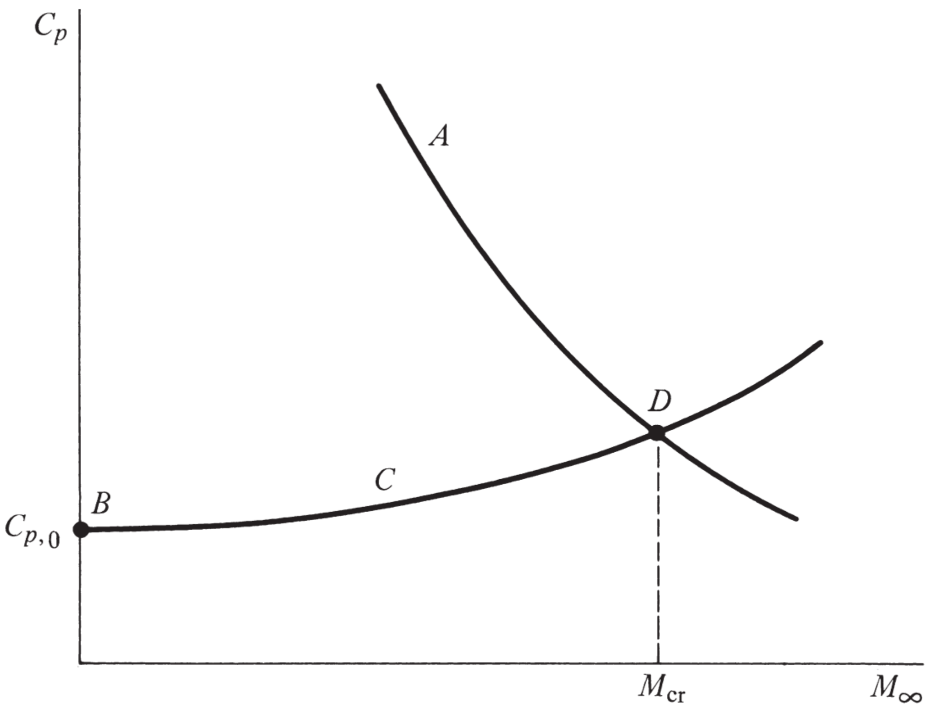
To make it clear, this expression allows you to calculate the pressure coefficient of the point of the flow where the local Mach number equals 1. It depends on the critical Mach number, which is the free-stream Mach number.

Keep in mind that the critical pressure coefficient is essentially the most negative value. The critical Mach number depends on the thickness of the airfoil and shape and is usually not known.

Estimation of the critical Mach number

What is the critical Mach number of your specific flight condition?

1. First obtain the $C_{p_{cr}} - M_{cr}$ curve. This curve tells you the critical Mach number and their corresponding critical pressure coefficient. But which one to choose?
2. Determine the C_{P_0} from the wind tunnel measurement. That is the pressure coefficient in which incompressible flow is considered, so $M < 0.3$. **the C_{p_0} is different for angle of attack and flight speed.**
3. Compute $C_p = \frac{C_{p_0}}{\sqrt{1-M_\infty^2}}$, this, essentially relates the effects of increasing Mach number to the pressure coefficient at the point where the critical Mach number happens. So, essentially, if u increase the free-stream Mach number at a point, pressure coefficient increases. Doing this, you will end up with the local Mach number equal to 1, the corresponding free-stream Mach number is the critical Mach number.
4. The intersection between the two curves gives the critical Mach number for the specific flight condition.



Equating both equations,

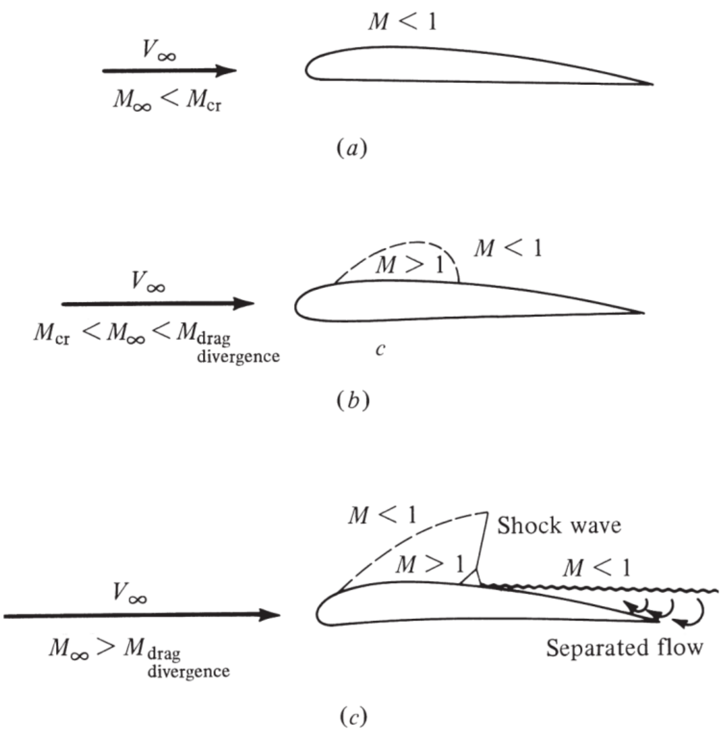
$$\frac{C_{p_0}}{\sqrt{1-M_{cr}^2}} = \frac{2}{M_{cr}^2}\left(\left(\frac{1+\frac{1}{2}(\gamma-1)M_{cr}^2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)$$

Usually, the location of the point where the sonic flow is reached, so the minimum pressure coefficient, is at $\frac{x}{c} = 0.3$.

Drag-Divergence Mach Number

Any free-stream Mach number above the critical Mach number will cause supersonic regions, creating a dramatic rise in the drag coefficient. Typically, the flow velocity at $0 < \frac{x}{c} < 0.45$, will be become supersonic. The free-stream Mach number at which c_d begins to increase rapidly is defined as the **drag-divergence Mach number,**

$$\boldsymbol{M_{cr} < M_{drag\ divergence} < 1.0}$$

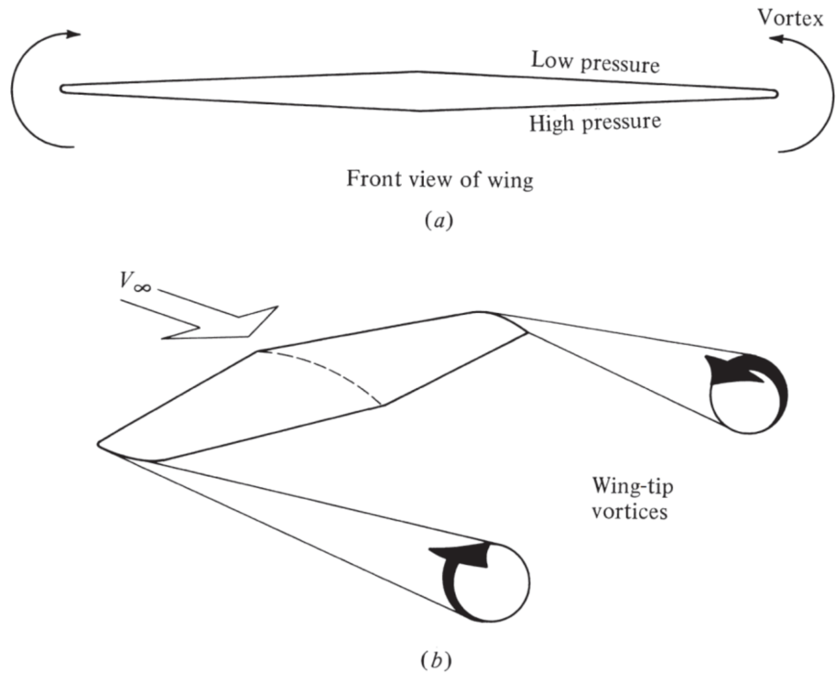


You can use $\boldsymbol{C_p = \frac{2}{M_\infty^2}\left(\left(\frac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)M^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)}$ to find the corresponding pressure coefficient.

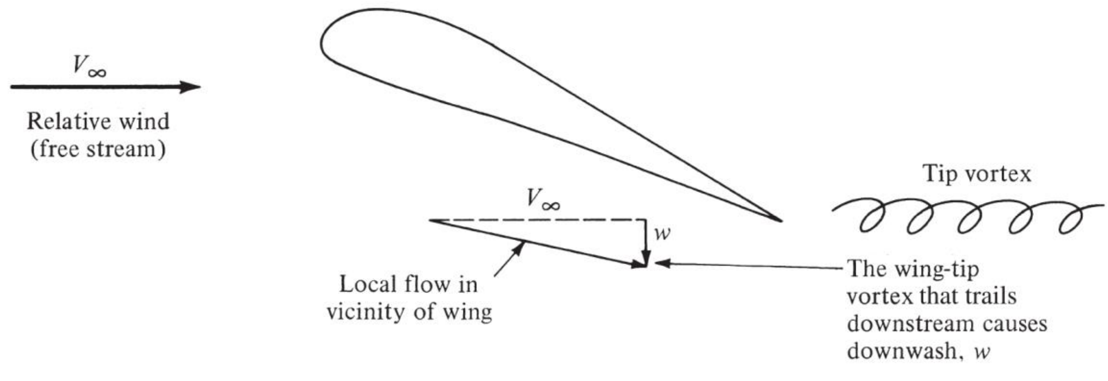
Lecture finite wings

Up until now, we have only considered aerodynamic properties of infinite wings. However, all wings have a tip, are finite. If u look at a $C_L - \alpha$ graph of an airfoil, are those values also true for the finite wing? No.

The fundamental differences between an airfoil and a finite wing is the at the end.



The wing generates lift by having a higher pressure distribution at the bottom than at the upper surface. Consequently, there is a tendency for the air to flow around the wing tips from the high-pressure side to the low-pressure side. The circular trailing motion is a **vortex.** These vortices downstream of the wing induce a small downward component of air velocity in the neighborhood of the wing itself. The vortices tend to drag the surrounding air around with them, inducing a velocity component downwards. This is called **downwash w.**



The local flow velocity in vicinity of the wing changes right now, consequently:

1. The angle of attack of the airfoil sections of the wing gets reduced in comparison to the angle of attack of the wing referenced to V_∞.
2. There is an increase in drag, **induced drag.** The wing tip vortices alter the flow field about the wing to change the surface pressure distributions in the direction of increased drag.

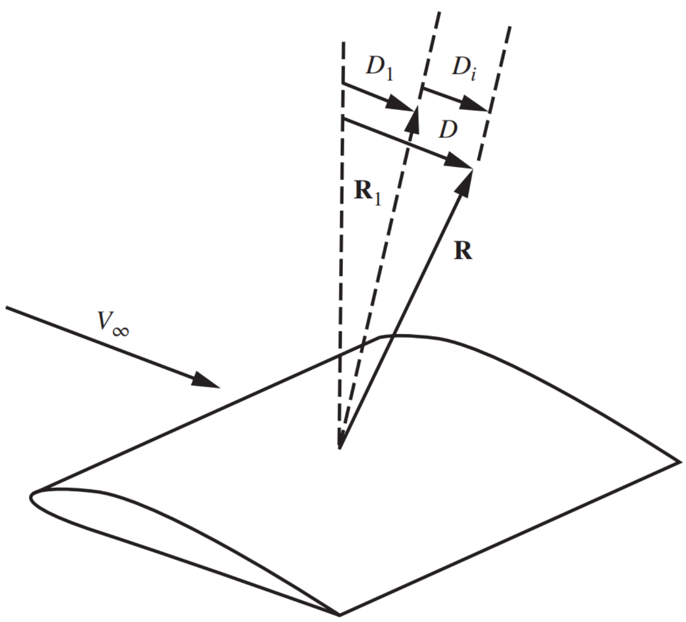
The C_L obtained from the pressure coefficient graph, is an airfoil lift coefficient, the actual lift coefficient is smaller,

$$C_{L_{wing}} < C_{L_{calculated\ from\ the\ pressure\ coefficient\ graph}}$$

The presence of induced drag for the finite wing adds to the already existing skin friction drag and pressure drag due to flow separation. $D_{profile} = D_{friction} + D_{flow\ separation}$. Hence

$$C_{D_{wing}} > C_{D_0}$$

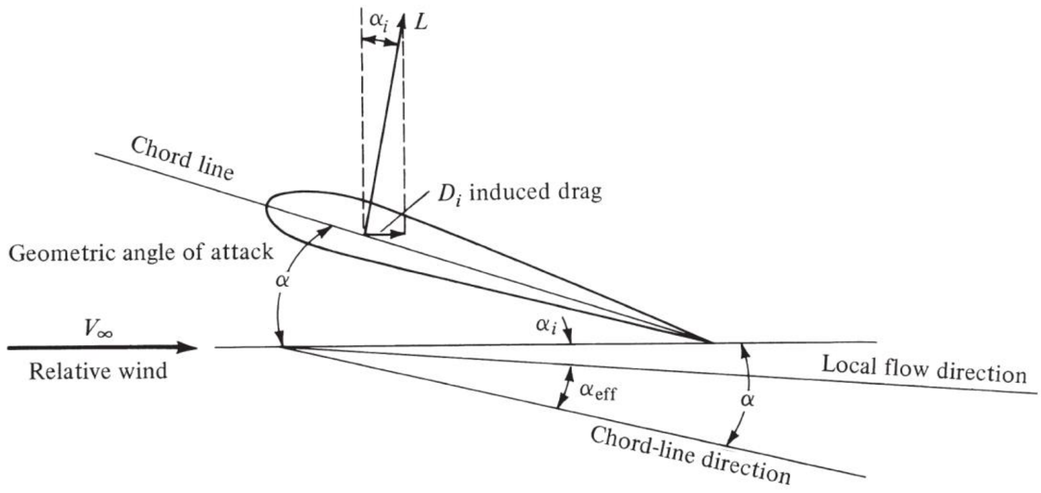
A way of conceptualizing induced drag can be seen below



$\boldsymbol{R_1}$ is the aerodynamic force generated in which no vortices were created. D_1 is the component parallel to the airspeed. $\boldsymbol{R}$ represents the actual resultant aerodynamic force, including the effect of wing-tip vortices. It changes the pressure distribution over the surface in such a fashion that $\boldsymbol{R}$ **is tilted backward,** causing $D > D_1$.

$$D_i = D - D_1$$

Induced drag is **pressure drag due to vortices**. To calculate induced drag, consider the following image. Here the angle between the airspeed and the mean chord line is the **geometric angle of attack α.** The presence of induced drag causes the local flow to deflect downwards



By angle α_i, **induced angle of attack.** The new angle of attack, the **effective angle of attack,** is less than the geometric angle of attack

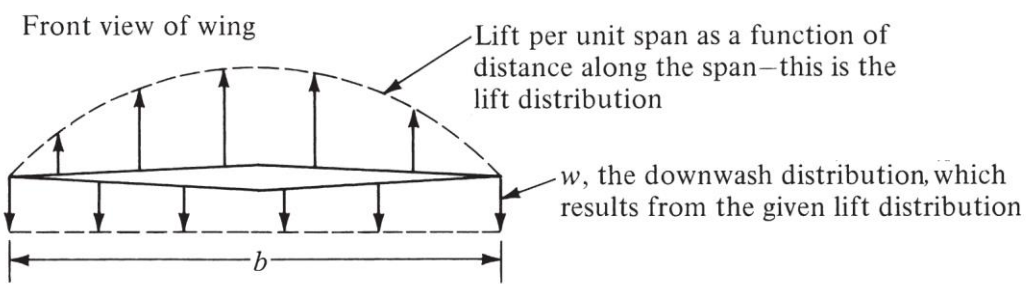
$$\alpha_{eff} = \alpha - \alpha_i$$

Because the angle of attack changes, and the lift always acts perpendicular to the flow velocity, the lift vector gets deflected by α_i. The tilted lift vector will contribute to a certain component of drag, parallel to airspeed, the induced drag D_i.

From geometry and small angle approximation,

$$D_i = L\alpha_i$$

The downwash causes a change in lift distribution to become elliptical.



You can see that because the distribution is elliptical, the local lift force varies over the wing, because of

1. The chord may vary in length
2. The wing may be twisted so that each airfoil section of the wing is at a different geometric angle of attack.
3. The shape of the airfoil may change along the span.

The elliptical lift distribution will cause a downwash distribution, changing the angle of attack by the induced angle of attack,

$$\alpha_i = \frac{C_L}{\pi AR}$$

Substituting

$$D_i = L\alpha_i = qS\frac{C_L^2}{\pi AR}$$

We can now define the **induced drag coefficient**

$$C_{D_i} = \frac{C_L^2}{\pi AR}$$

This expression holds for an elliptical distribution. In reality, such distributions often deviate from a perfect elliptical shape. Therefore, a **span efficiency factor** $\boldsymbol{e}$ is defined such that

$$\boldsymbol{C_{D_i} = \frac{C_L^2}{\pi ARe}}$$

For elliptical distribution, $e = 1$. From this, high aspect ratios, so a large span and a small chord, will cause the least induced drag. Induced drag will cause the most drag at speeds below stalling. The total drag becomes:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi ARe}$$

Here,

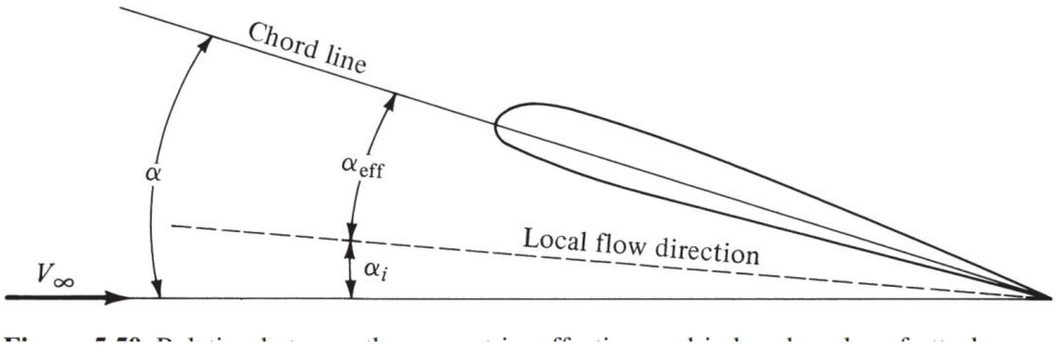
$$C_{D_0} = C_{D_{friction}} + C_{D_{flow\ separation}}$$

The C_{D_0} is the profile drag, or the total drag from an airfoil, this is the drag coefficient that can be obtained from $C_D - \alpha$ curves.

Change in the lift slope

Finite wings will differ in two major respects from infinite wings when it comes to the lift curve. The first one was already discussed, addition of induced drag.

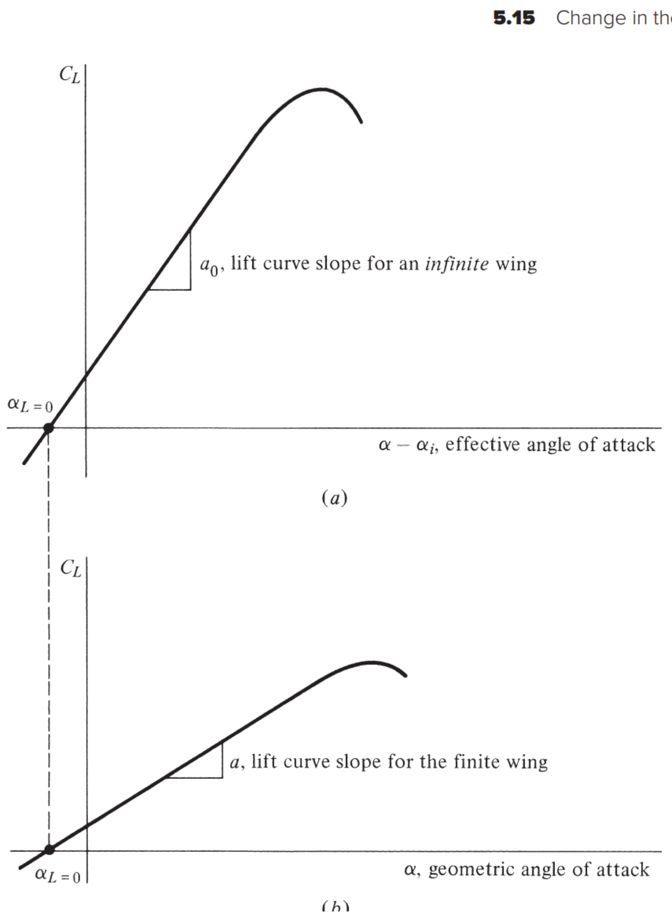
The second difference is that the lift curve for a finite wing with the same airfoil cross section has a smaller slope than for an infinite wing. Recall the due to the downwash, the effective angle of attack is less than the geometric angle of attack.



The induced angle of attack could be calculated using,

$$\alpha_i = \frac{C_L}{\pi AR}$$

The flow over a finite wing at an angle of attack is essentially the same as the flow over an infinite wing at an angle of attack α_{eff}, the only difference being the downwash.



Here the $C_L - \alpha_{eff}$ is the exact same curve as for a $C_L - \alpha$ curve for an infinite wing. The only problem is that the effective angle of attack cannot be seen from naked eyes, it is just a theoretical angle. It, therefore makes much more sense to plot $C_L - \alpha$ for a finite wing. This results in a smaller slope, hence

$$\left(\frac{dC_L}{d\alpha}\right)_{finite} < \left(\frac{dC_L}{d\alpha}\right)_{infinite}$$

For an infinite wing,

$$\frac{dC_L}{d\alpha} = 2\pi$$

Note that at zero lift, there is no induced drag, so $\alpha_{eff} = \alpha = 0$. Imagine if we know a_0 for an infinite wing, what is a for a finite wing with a given aspect ratio?

We know that, because $C_L - \alpha_{eff}$ is the exact same curve as for a $C_L - \alpha$ curve for an infinite wing,

$$\frac{dC_L}{d(\alpha - \alpha_i)} = a_0$$

Integrating, we find

$$C_L = a_0(\alpha - \alpha_i)$$

Knowing that

$$\alpha_i = \frac{57.3C_L}{\pi ARe}$$

With α_i in degrees

$$C_L = a_0\left(\alpha - \frac{57.5C_L}{\pi ARe}\right)$$

Solving for C_L and differentiating w.r.t α,

$$\left(\frac{dC_L}{d\alpha}\right)_{finite} = \frac{a_0}{1 + \frac{57.3a_0}{\pi eAR}}$$

This gives us the lift slope for a finite wing. a_0 is the lift slope of a finite wing with zero lift, which can be found from a $C_D - \alpha$ curve. You can see that $a < a_0$, proving that the slope of a finite wing is smaller.

In short, finite wings cause,

1. Induced drag: $C_D = c_d(profile\ drag) + \frac{C_L^2}{\pi ARe}$
2. Change in lift slope →

$$\left(\frac{dC_L}{d\alpha}\right)_{finite} = \frac{a_0}{1 + \frac{57.3a_0}{\pi eAR}}$$

Tips Aerodynamics

- The aerodynamic force caused by a flow is due to pressure distribution (lift and separated drag), but also without pressure distribution (skin friction).
- At supersonic speeds, the pilot doesn't hear the shock explosion, because the sound waves of the shock are smaller than the airspeed of the aircraft.
- De laval = supersonic nozzle
- Transonic flow is flying at the critical Mach number.
- Pitot tube = total pressure, Pitot static tube = static pressure

- Angle of incidence = geometric angle of attack
- Supersonic nozzle, at the throat, M = 1.
- High critical airfoils have a thinner upper surface. The lower surface doesn't have an effect, because the shock waves appear on the upper surface.
- Drag coefficient of the airfoil = zero lift drag coefficient (since for an airfoil, not vortices). $C_{Dair} = C_{Dwing} - \frac{C_L^2}{\pi Ae}$
- $p_\infty = p_{tot} - \frac{1}{2}\rho V_\infty^2$, $p_{stat,local} = p_{tot} - \frac{1}{2}\rho V_{local}^2$, $p_{tot} = p_{am} + \frac{1}{2}\rho V_\infty^2$
- $p_\infty = p_{tot} - \frac{1}{2}\rho V_\infty^2 = p_{am} + \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V_\infty^2 = p_{amb}$, $T_\infty = T_{amb}$, $\rho_\infty = \rho_{amb}$
- Ambient conditions are the same as free stream conditions.
- $\frac{T_{st}}{T_\infty(=T_{amb})} = 1 + \frac{\gamma-1}{2}M_\infty^2$
- If information is given about a point on the flow where $V = 0$, $\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2$. If temperature is given, then $c_pT_\infty + \frac{1}{2}V_\infty^2 = c_pT_A + \frac{1}{2}V_A^2$, where A = point on airfoil.
- Airfoil to finite wing calculations:
- To calculate skin friction drag, multiply the coefficient by two for two wings, $D_f = 2C_f q_\infty S$
- Kinematic viscosity = $v = \frac{\mu}{\rho} \rightarrow Re = \frac{\rho Vc}{\mu} = \frac{Vc}{v}$
- The area-velocity relation for compressible flows requires the isentropic hypothesis
- True airspeed = free stream velocity
- There are three points you can use: free stream point, stagnation point, point outside boundary at airfoil. For isentropic flow relation, you can use these points. For the isentropic reservoir formula, you have to use stagnation point. When given a point A outside boundary layer, first try to use energy equation.
- Euler equation assumptions:
  a) no friction
  b) no gravity
  c) steady flow
- The local Mach number reaches 1 at the maximum speed, thus $C_{Pmin} = C_{P0}$ has to be used for the determination of critical mach number. If C_{Pmin} occurs

at V_{max}. Use $C_P = \frac{C_{pmin}}{\sqrt{1-M^2}}$. Draw this in the $C_P - M$-curve and look at the intersection.

- Be aware that if CPo decreases, it becomes more negative. Thus, critical pressure coefficient increases as critical Mach number increases.
- $M_{crse} = \frac{M_{cr}}{\cos(\beta)} \rightarrow \frac{M_{Cr,swe}}{M_{cr}} = \frac{1}{\cos\beta} = 1.15\ (example) \rightarrow$ increase of 15%.
- Increasing the skin friction, the flow would become more turbulent, as the transition from laminar to turbulent is increased.
- Separation at the trailing edge causes the lift coefficient to deviate from the linear relation with angle of attack.
- Streamlining means increasing velocity.
- Pressure drag contributes to flow separation, but also shock waves at high Mach numbers, because at high speeds the pressure becomes low, resulting in $M_{local} > 1$.
- Induced drag influence summary:

From Airfoil to Wing	**From Wing to Airfoil**
Determine $\left[\frac{dC_L}{d\alpha}\right]_{airfoil} = a_{air}$	Determine $\left[\frac{dC_L}{d\alpha}\right]_{airfoil} = a_w$
The C_{Lwing} at $\alpha = \alpha_{air}$ will be $C_{Lwing} = a_{air}(\alpha_{air} - \alpha_{CL=0} - \alpha_i)$ $C_{Lwi} = a_{air}\left(\alpha_{air} - \alpha_{CL=0} - \frac{57.3C_{Lwing}}{\pi Ae}\right)$	The C_{Lairf} at $\alpha = \alpha_{wing}$ will be $C_{Lairf} = a_w(\alpha_{wing} - \alpha_{CL=0} + \alpha_i)$ $C_{Lairf} = a_w\left(\alpha_{wing} - \alpha_{CL=0} + \frac{57.3C_{Lairf}}{\pi Ae}\right)$
Solving the above equation: The C_{Lwing} at $\alpha = \alpha_{air}$ will be $C_{Lw} = \frac{a_{air}}{1 + \frac{57.3a_{air}}{\pi Ae}}(\alpha_{air} - \alpha_{CL=0})$	Solving the above equation: The C_{Lairf} at $\alpha = \alpha_{wing}$ will be $C_{Lairf} = \frac{a_w}{1 - \frac{57.3a_w}{\pi Ae}}(\alpha_{wing} - \alpha_{CL=0})$
Determine $\left[\frac{dC_L}{d\alpha}\right]_{wing} = \frac{a_{air}}{1 + \frac{57.3a_{air}}{\pi Ae}}$	Determine $\left[\frac{dC_L}{d\alpha}\right]_{airf} = \frac{a_w}{1 - \frac{57.3a_w}{\pi Ae}}$

- $C_{Dwing} = C_{D_{air}} + \frac{C_{Lwing}^2}{\pi Ae}$